Name: Solutions

Math 206: Fall 2013
Exam 2: November 1

calculator allowed

Correct answers accompanied by incorrect or incomplete work will not receive full credit.
Good Luck!

1. (3 points each) Determine whether each of the following statements is *true* or *false*. No justification necessary, no partial credit available.

   (a) If $f_x(0,0)$ and $f_y(0,0)$ exist, then f must be differentiable at $(0,0)$.

   False

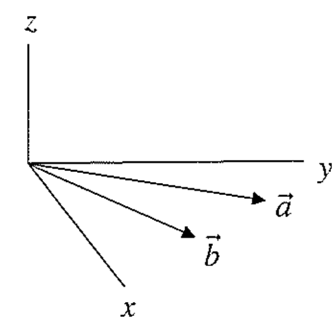
   (b) A partial derivative is a specific example of a directional derivative.

   True

   (c) $\nabla f(a,b)$ is perpendicular to the graph of $z = f(x,y)$ at the point (a,b).

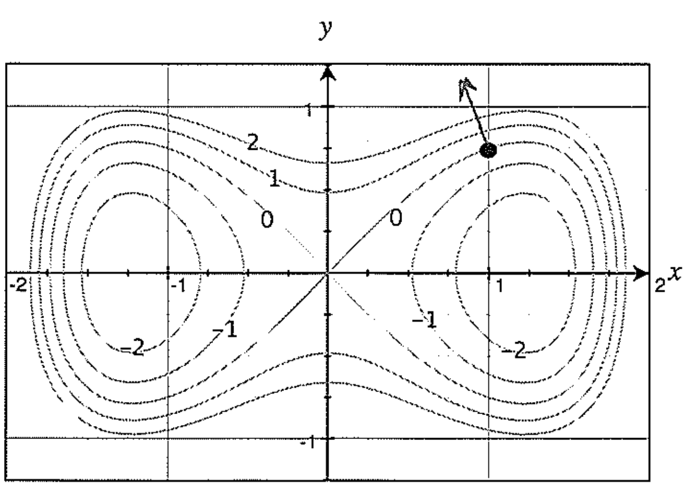
   False

   (d) Vectors $\vec{a}$ and $\vec{b}$ are as pictured in the diagram, both are in the xy-plane.
   True/False: $\vec{a} \times \vec{b} = t\hat{k}$, where t is a positive number.



   False

2. (5 points) Consider the function $g(x,y)$ with contour diagram below. Sketch a vector that points in the direction of $\nabla g(1, 0.75)$.



3. (8 points) The consumption of beef by one household, C (in pounds per week) is given by the function $C = f(I, p)$, where I is the annual household income in thousands of dollars, and p is the price of beef in dollars per pound. Explain the meaning of the statement: $f_I(80, 3) = 0.022$, include units in your answer. For a household w/ annual income of \$80,000 when the price of beef is \$3 per lb, if the annual income is increasing, then the consumption of beef is increasing at a rate of 0.022 pounds per week / \$. Alternately: a thousand \$ increase in annual income results in an approximate increase in consumption of 0.022 pounds per week.

4. (12 points) The table of some values of $C = f(I, p)$ is given below. Find a linearization of f at $(40, 3)$.

		Price of beef, p (\$/lb)			
		3	3.5	4	4.5
Household income per year, I (\$1000)	20	2.65	2.59	2.51	2.43
	40	4.14	4.05	3.94	3.88
	60	5.11	5.00	4.97	4.84
	80	5.35	5.29	5.19	5.07

$$f_p(40,3) \approx \frac{4.05 - 4.14}{3.5 - 3} = -.18$$

$$f_I(40,3) \approx \frac{5.11 - 4.14}{60 - 40} = .0485$$

$$\boxed{f(I,P) \approx 4.14 - .18(p-3) + .0485(I-40)}$$

5. (15 points) Find the equation of the plane tangent to $z^2 + 2zy + 4y = x^2 + 3$ at the point $(3, 1, -4)$.

the given surface is a level set to

$T(x,y,z) = z^2 + 2zy + 4y - x^2$

$T_x = -2x \longrightarrow T_x(3,1,-4) = -6$

$T_y = 2z + 4 \rightarrow T_y(3,1,-4) = -4$

$T_z = 2z + 2y \rightarrow T_z(3,1,-4) = -6$

$\nabla T(3,1,-4) = -6\hat{\imath} - 4\hat{\jmath} - 6\hat{k}$ is $\perp$ to level surface and the tangent plane. So we use this to write the equation of the plane

$$\boxed{-6(x-3) - 4(y-1) - 6(z+4) = 0}$$

6. (15 points) Let $w = 3x\cos y$. If $x = u^2 + v^2$ and $y = \dfrac{v}{u}$, find $\dfrac{\partial w}{\partial u}$ at the point $(u, v) = (2, 3)$. Give your answer to 2 decimal places. (Set your calculator to radians.)

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= (3\cos y)(2u) + 3x(-\sin y)\left(\frac{-v}{u^2}\right)$$

when $(u,v) = (2,3)$ $(x,y) = (13, 3/2)$

$$\left.\frac{\partial w}{\partial u}\right|_{(u,v)=(2,3)} = (3)(\cos\tfrac{3}{2})(4) + 3(13)(\sin\tfrac{3}{2})\left(\tfrac{3}{4}\right)$$

$$= .849 + 29.177 = \boxed{30.03}$$

7. (12 points) Suppose that $f_x(x, y) = \frac{1}{2}(x + 2y)^{-1/2}$ and $f_y(x, y) = (x + 2y)^{-1/2}$. Also suppose that $f(1, 0) = 1$. Find the quadratic Taylor polynomial (i.e., the quadratic approximation) of $f(x, y)$ at $(1, 0)$.

$f_{xx} = -\frac{1}{4}(x+2y)^{-3/2}$, $\quad f_{xy} = -\frac{1}{4}(x+2y)^{-3/2}(2)$

$f_{yy} = -\frac{1}{2}(x+2y)^{-3/2}(2)$

$f_x(1,0) = \frac{1}{2}$, $\quad f_y(1,0) = 1$, $\quad f_{xx}(1,0) = -\frac{1}{4}$

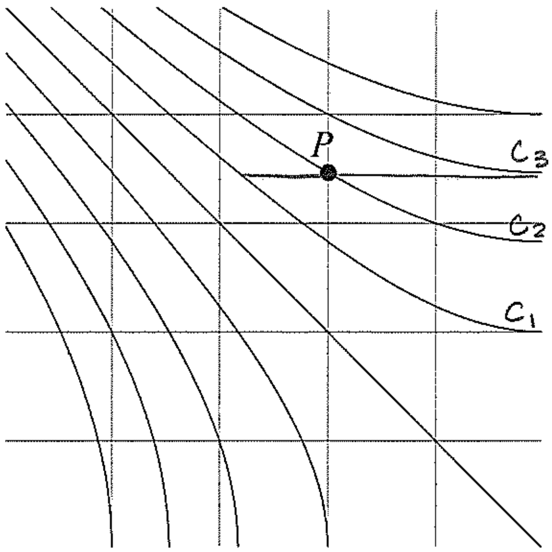
$f_{xy}(1,0) = -\frac{1}{2}$, $\quad f_{yy}(1,0) = -1$

$$Q(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + \frac{f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b) + f_{yy}(a,b)(y-b)^2}{2}$$

where $(a,b) = (1,0)$ in this problem.

so $\boxed{Q(x,y) = 1 + \frac{1}{2}(x-1) + y - \frac{1}{8}(x-1)^2 - \frac{1}{2}(x-1)y - \frac{1}{2}y^2}$

8. (10 points) Using the contour diagram for $f(x, y)$, find the sign of $f_{yy}(P)$ given that $f_{xx}(P) < 0$. Justify your answer.



$f_{xx}(P) < 0$ means the graph is concave down in the x-direction. So near P the cross section w/ y fixed is either or .

From the contour curves we see that slope is steeper to the left of P (b/c the contours are closer together), thus we have . This tells us that the heights of the contour curves have the relation $C_1 < C_2 < C_3$.

Then we know $f_y(P) > 0$. As y increases near P, the slope in the y-direction gets steeper. So f_y near P is increasing as y increases. Thus $\boxed{f_{yy}(P) > 0.}$

9. (8 points) Select ALL the planes that could NOT be tangent planes to the graph of a function $f(x, y) = z$ that is differentiable everywhere. (Briefly justify your choices.)

(a) $3x - 5y - z = 2$
(b) $5x + 3y = 2$
(c) $3x + 5y = 2$
(d) $3x + 5y + z = 2$
(e) $5y = 2$

If a fcn is differentiable everywhere then all tangent planes have the form

$$z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

b, c, and e do not contain "z" they don't have the required form.

10. (3 points) What is your favorite food? risotto