

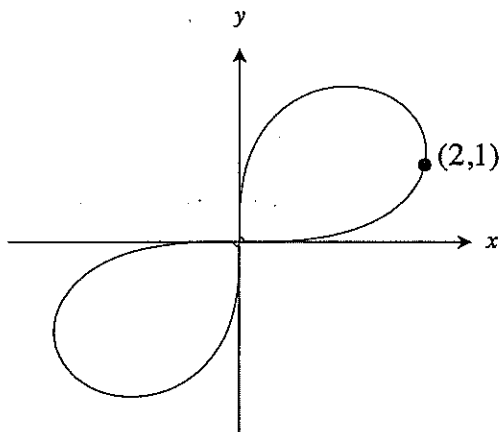
Name: Solutions

Math 105: Fall 2013

Quiz 6: November 1

Good Luck!

1. The graph of $2(x^2 + y^2)^2 = 25xy$ is shown below (graph not to scale). Use implicit differentiation to compute the slope of the line tangent to the graph at (2, 1).



$$\frac{d}{dx} (2(x^2 + y^2)^2 = 25xy)$$

$$2(2)(x^2 + y^2)(2x + 2y \frac{dy}{dx}) = 25y + 25x \frac{dy}{dx}$$

we want $\frac{dy}{dx}$ at (2, 1) so we'll

plug in $x=2$, $y=1$ then solve for $\frac{dy}{dx}$

$$2(2)(4+1)(4 + 2 \frac{dy}{dx}) = 25 + 50 \frac{dy}{dx}$$

$$20(4 + 2 \frac{dy}{dx}) = 25 + 50 \frac{dy}{dx}$$

$$80 + 40 \frac{dy}{dx} = 25 + 50 \frac{dy}{dx}$$

$$55 = 10 \frac{dy}{dx}$$

$$\boxed{5.5 = \frac{dy}{dx}}$$

solving for $\frac{dy}{dx}$ first

$$(x^2 + y^2) [8x + 8y \frac{dy}{dx}] = 25y + 25x \frac{dy}{dx}$$

$$8x(x^2 + y^2) + 8y(x^2 + y^2) \frac{dy}{dx} = 25y + 25x \frac{dy}{dx}$$

$$8x^3 + 8xy^2 - 25y = 25x \frac{dy}{dx} - (8x^2y + 8y^3) \frac{dy}{dx}$$

$$\frac{8x^3 + 8xy^2 - 25y}{25x - 8x^2y - 8y^3} = \frac{dy}{dx} \implies \text{plug in } (2, 1) \implies \frac{dy}{dx} = \frac{8(8) + 8(2) - 25}{50 - 8(4) - 8} = \frac{55}{10} = \boxed{5.5}$$

OVER

2. Let $f(x) = \arcsin(2x+1)$.

(a) Find $f\left(\frac{-3}{4}\right)$. Express your answer exactly, not as a decimal approximation.

$$f\left(\frac{-3}{4}\right) = \arcsin\left(2\left(\frac{-3}{4}\right)+1\right) = \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

(b) Find $f'\left(\frac{-3}{4}\right)$. Express your answer as a decimal approximation.

$$f(x) = \arcsin(2x+1)$$

$$f'(x) = \frac{1}{\sqrt{1-(2x+1)^2}} (2x+1)' = \frac{1}{\sqrt{1-(2x+1)^2}} \cdot 2$$

$$f'\left(-\frac{3}{4}\right) = \frac{2}{\sqrt{1-\left(-\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{\frac{3}{4}}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}} \approx 2,3094$$

