Math 106D **Test 2** **Name:**_____

Show all steps.
You do not need to simplify your arithmetic.

Evaluate any THREE of the following four integrals. Below, mark which three you are doing.

____ **1.** (15 pts.) $\int \sin^4 x \cos^3 x \, dx$

____ **2.** (15 pts.) $\int \frac{x+2}{(x-5)(x-1)} \, dx$

____ **3.** (15 pts.) $\int x \sin\left(\frac{x}{4}\right) dx$

____ **4.** (15 pts.) $\int \frac{1}{(x^2+4)^{3/2}} \, dx$

5. (18 pts.) (a) Find the second-order Taylor polynomial P_2 with base point $x_0 = 5$ for the function $f(x) = 1/x$.

(b) Apply our usual error formula to find a bound on the size of the error in using P_2 to approximate f on the interval $[3, 7]$.

6. (12 pts.) Use an antiderivative to determine the convergence/divergence of the improper integral $\int_1^\infty \frac{1}{x} - \frac{3 - 100x^{-101}}{3x + x^{-100}} \, dx$. If the integral converges, evaluate it. (Hint: The integral is easier than it looks at first. Notice that in the second fraction, the numerator and denominator have a certain relationship to each other!)

7. (8 pts.) Use the comparison test to determine the convergence/divergence of the improper integral $\int_2^\infty \frac{1}{\sqrt{x} + x^3} \, dx$. Clearly indicate the comparison you are using, and whether it's an underestimate or overestimate. (Make sure to use an estimate appropriate for the conclusion that you reach.)

8. (16 pts.) The lifespans of bacteria of the species *E. coliflour* are distributed according to the density function $f(x) = \begin{cases} 4x^3 & \text{if } 0 \le x \le 1; \\ 0 & \text{otherwise.} \end{cases}$

(a) What fraction of *E. coliflour* will have a lifespan between 0.1 and 0.2?

(b) Find the mean lifespan.