Math 205 Quiz 5
Name:

1. Let $\beta = \{1, 2-t, t^2+t^3, t^3\}$ be a basis for $\mathbb{P}_3$. Suppose you are given the following coordinate vector:

$$[\vec{p}(t)]_\beta = \begin{bmatrix} 2 \\ 1 \\ 3 \\ -1 \end{bmatrix}$$

What is $\vec{p}(t)$?

$$2*1 + 1*(2-t) + 3*(t^2+t^3) - 1*t^3 = 4 - t + 3t^2 + 2t^3$$

2. Let $\Delta = \{1+t, 2t, t+t^2, t^3\}$ be a basis for $\mathbb{P}_3$. What is the coordinate vector for the vector $\vec{p}(t) = 1 + 4t + 3t^2$?

$$\vec{p}(t) = 1 + 4t + 3t^2 = 1*(1+t) + 0*2t + 3*(t+t^2) + 0*t^3$$

$$[\vec{p}(t)]_\beta = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

3. Determine whether the vectors $\{1+2t^2, 4+t, t+5t^2\}$ in $\mathbb{P}_2$ are linearly independent or linearly dependent by using a coordinate vector argument.

   Use the ordered basis $\{1, t, t^2\}$ to determine the coordinate vectors.

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 5 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   There is a pivot in every column, so these vectors are linearly independent.

4. $B = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 0 & -2 & 4 & 8 \\ 12 & 18 & 8 & 4 \end{bmatrix}$

$$\text{rref}(B) = \begin{bmatrix} 1 & 0 & 0 & 1/21 \\ 0 & 1 & 0 & -4/7 \\ 0 & 0 & 1 & 12/7 \end{bmatrix}$$

   (a) What is the dimension of the Col(B)? How do you know?
   Three dimensional. There are three pivot columns.

   (b) What is the dimension of the Nul(B)? How do you know?
   One dimensional. There is one free variable.

   (c) The Nul(B) is a subspace of $\mathbb{R}^k$. What is k? $k = 4$

   (d) The Col(B) is a subspace of $\mathbb{R}^p$. What is p? $p = 3$