1. Find $\int \frac{2\,x^3 + 10\,x^2 + 24\,x + 19}{x^2 + 3x + 2}\,dx$ using partial fractions.

2. Find $\int \tan^3 t\ \sec t\ dt$; show all your work.

3. Use an appropriate trig substitution to find $\int \frac{dx}{\left(\sqrt{4-x^2}\,\right)^3}$. This integral can also be written $\int \frac{dx}{(4-x^2)^{3/2}}$. Show all your work, including any triangles you use to "unsubstitute".

4. The integral $\displaystyle\int_8^9 \frac{1}{\sqrt[5]{x-8}}\,dx$ is improper. Use antiderivatives to determine if the interval converges or diverges. If the integral converges find out to what. Support your answer with a useful table. Full credit for this problem requires showing all necessary work and use of correct notation.

5. Find $\displaystyle\int 2x(\arctan x)\,dx$ using integration by parts. Near the end, you might this useful: $\dfrac{x^2}{x^2+1}$ is $1-\dfrac{1}{x^2+1}$.

6. Let $f(x) = \dfrac{9x^2+1}{9\,x^6+6\,x^4+x^2}$. Consider $\int_1^\infty f(x)\ dx$.

6A: For large values of x, $f(x)$ "looks like" $1/x^4$. Show how $g(x) = 1/x^4$ can be used to determine that $\int_1^\infty f(x)\ dx$ converges by correct use of the comparison test.

6B: Find $\int_1^{10} f(x)\ dx$ using your calculator's built-in integration (as we've done in class). Write your results here to as many places as the calculator shows.

6C: Hopefully most of the area under the graph of $f(x)$ from 1 to ∞ is actually between $x = 1$ and $x = 10$ so that the answer to 6B can be used as an approximation to $\int_1^\infty f(x)\ dx$. In fact, show how $g(x)$ is used to estimate, at most, how much area we are leaving out when we discard the "tail" $\int_{10}^\infty f(x)\ dx$. You may use the fact that when $p > 1$, $\int_m^\infty \dfrac{1}{(x-c)^p}\ dx$ converges to $\dfrac{1}{(p-1)(m-c)^{p-1}}$.

7: BONUS. It turns out that $f(x) = \dfrac{9x^2+1}{9\,x^6+6\,x^4+x^2}$ has an easy-to-find antiderivative: The denominator is $(3x^3+x)^2$, so that $\int f(x)\,dx$ has the form $\int \dfrac{du}{u^2}$.

With this, you can find $\displaystyle\int_1^{10} \frac{9x^2+1}{9\,x^6+6\,x^4+x^2}\,dx$ and $\displaystyle\int_1^{\infty} \frac{9x^2+1}{9\,x^6+6\,x^4+x^2}\,dx$ directly using the antiderivative — no numerical approximations needed! Indeed, find both integrals using the accepted techniques!

Then compare your answer to $\displaystyle\int_1^{10} \frac{9x^2+1}{9\,x^6+6\,x^4+x^2}\,dx$ in problem 6B, and also verify that that amount differs from the "whole area" by no more than the amount "guaranteed in 6C.

8: Consider $h(x) = \dfrac{9\,x^2+100\,x}{9\,x^6+50}$. This function looks like $1/x^4$ as x gets larger and larger. So you would expect $\int_1^\infty h(x)\,dx$ to converge. However, comparing $h(x)$ to $1/x^4$ won't work, because $h(x)$ is not less than $1/x^4$ for $x \geq 1$. Find the smallest integer A for which $h(x) \leq A/x^4$. Show your work; sketch and label any graphs you may have plotted on your calculator to help with this problem or otherwise explain how you found A.