Math 106 Fall 2014 Exam 2.2 October 31, 2014

Directions: Complete all of the following to the best of your ability. If you do not understand a question, please let me know; I may be able to assist you. SHOW ALL WORK! You will be graded primarily on the method you use, not your final answer. GOOD LUCK!

Name: _____

Question	Points Possible	Score
1	20	
2	16	
3	14	
4	12	
5	13	
Total	75	

DO NOT WRITE IN THE BOXES ABOVE!

Each question is broken up across two pages. You can use any space to answer any part. Be sure to label your work so it can be graded appropriately.

1. The following questions have to do with the integral $\int_1^\infty \frac{\ln(x)}{x^2}\,\mathrm{d}x$

   (a) Evaluate $\int \frac{\ln(x)}{x^2}\,\mathrm{d}x$

(b) Suppose your answer from part (a) is $\dfrac{\ln x - 2x}{12x^2}$. Explain how to use this answer to understand if the integral $\displaystyle\int_1^\infty \frac{\ln(x)}{x^2}\,\mathrm{d}x$ is convergent or divergent.

(c) Now suppose that $\dfrac{7}{x^{4/3}} \leq \dfrac{\ln(x)}{x^2} \leq \dfrac{9}{x^{3/2}}$ over some interval $[a, \infty)$.

Use this idea along with the Comparison Test to determine if $\displaystyle\int_a^\infty \frac{\ln(x)}{x^2}\,\mathrm{d}x$ is convergent or divergent. Remember, your answer must include sentences!

2. Consider the following integral. $\displaystyle\int \frac{1}{4\sqrt{x+1}+x+4}\,\mathrm{d}x$

(a) Use a rationalizing substitution and related steps to convert the integral to a Case I or Case II Partial Fractions result. Your final answer for this part should be an integral of a rational function. **DO NOT SOLVE FOR COEFFICIENTS AND DO NOT INTEGRATE!**

(b) Suppose your result from part (a) is $\displaystyle\int \frac{8u}{(u-1)^2(u+1)}\,\mathrm{d}u$. Evaluate this new integral.

3. Consider the following integral. $\int \sin^3(x)\cos^4(x)\ \mathrm{d}x$

(a) Evaluate $\int \sin^3(x)\cos^4(x)\ \mathrm{d}x$

(b) The probability that Bailey falls asleep during a class within x minutes after the start is a given by the function

$$f(x) = \begin{cases} c \cdot \sin^3(x)\cos^4(x) & \text{if} \quad 0 \leq x \leq \pi \\ 0 & \text{otherwise.} \end{cases}$$

where c is some constant number.

Use properties of a pdf (probability density function) and your answer from part (a) to find the value for c.

4. The following questions have to do with the integral $I = \int \sin(3x) \cdot \cos(x)\ \mathrm{d}x$.

   (a) Demonstrate the results of one application of integration by parts applied to $\int \sin(3x) \cdot \cos(x)\ \mathrm{d}x$.

(b) Continue your work from part (a) and evaluate $I = \int \sin(3x) \cdot \cos(x)\ \mathrm{d}x$.

5. Consider the limit $\lim_{x \to 0} \dfrac{x^2}{e^{3x} - 3x - 1}$

   (a) Find a relevant Taylor Polynomial that will (eventually) allow you to evaluate the limit. For this part, focus on calculating and assembling the pieces of your Taylor Polynomial.

(b) Use your Taylor Polynomial answer to now evaluate the limit

$$\lim_{x \to 0} \frac{x^2}{e^{3x} - 3x - 1}.$$