

**MATH 205A,B - LINEAR ALGEBRA  
WINTER 2015**

QUIZ 6

**NAME:** \_\_\_\_\_ **Section:**(Circle one)    A(8 : 00)    B(9 : 30)

Show **ALL** your work **CAREFULLY**.

Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 3 & 3 \\ 0 & 1 & -1 \end{bmatrix}.$$

(a) Find an explicit description of the null space  $\text{Nul}A$ , by listing vectors that span the null space.

**First,  $\text{Nul}A = \{\vec{x} \mid A\vec{x} = \vec{0}\}$  so that  $\text{Nul}A$  is simply the set of all solutions to the homogeneous system  $A\vec{x} = \vec{0}$ .**

Now,

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 3 & 3 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 3 & 3 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 2 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

It follows that  $x_1 + 3x_3 = 0$  and  $x_2 - x_3 = 0$  thus the solutions are

$$\text{Nul}A = \left\{ \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} \mid x_3 \in \mathbb{R} \right\}$$

so  $\text{Nul}A$  is a line.

(b) Find a basis for the column space  $\text{Col}A$ .

Based upon the calculations in (a), the first two columns have pivots. Thus,

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \right\}$$

is a basis for  $\text{Col}A$ .