Math 205 A B Quiz 05 blue page 1 10/26/2012 Name suggested soln

1. The consumption matrix C for an economy with three sectors G, H and M and the final demand vector $\mathbf{d}$ of the open sector are $C = \begin{bmatrix} 0.02 & 0.1 & 0.01 \\ 0.01 & 0.2 & 0.05 \\ 0.03 & 0.4 & 0.07 \end{bmatrix}$ and $\mathbf{d} = \begin{bmatrix} 400 \\ 500 \\ 600 \end{bmatrix}$, respectively.

1A) Find $\mathbf{x}$, the vector showing the total numbers of units of goods produced by the three sectors G, H and M. Show all your work and your answer rounded to *TWO* digits after the decimal point.

We know that $\vec{x} = C\vec{x} + \vec{d}$

or, $(I_3 - C)\vec{x} = \vec{d}$

there are at least two ways to proceed:

① Find $RREF([I_3 - C \mid \vec{d}])$

② Find $\vec{x} = (I_3 - C)^{-1}\vec{d}$

either method produces

$$\vec{x} = \begin{bmatrix} 488.45 \\ 690.99 \\ 958.12 \end{bmatrix} \begin{array}{l} \leftarrow G \\ \leftarrow H \\ \leftarrow M \end{array}$$

1B) Each unit produced by H requires how many units of G's product? 0.1

1C) Of the total number of units produced by M, how many are consumed by H?

(0.4 units of M required per unit of H made) $*$ (690.99 units of H made) =

$0.4 * 690.99$ units of M

$\boxed{= 276.40 \text{ units of } M}$

(the answer has nothing to do with 958.12!)

2. Let $C = \begin{bmatrix} 2 & 1 & 3 & 4 & 1 \\ 4 & 3 & 5 & 6 & 7 \\ -8 & -1 & -15 & -22 & 14 \end{bmatrix}$, then the RREF of C is $\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$.

Label the columns of C as $\mathbf{c}_1$, $\mathbf{c}_2$, ...

2A) Find a basis for Col(C). Don't write the vectors out; use the names $\mathbf{c}_1$, etc.

The RREF of C tells us the pivot columns of C form a basis $\therefore \{\vec{c}_1, \vec{c}_2, \vec{c}_5\}$

2B) Find the sum $\mathbf{s}$ of the last three column vectors of C. Now, $\mathbf{s}$ must be in Col(C). Indeed, express $\mathbf{s}$ as a LC of the basis vectors from part 2A. Show any matrices (augmented, RREF'd, etc) involved in your work.

So $\vec{s} = \vec{c}_3 + \vec{c}_4 + \vec{c}_5 = \begin{bmatrix} 3+4+1 \\ 5+6+7 \\ -15-22+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \\ -23 \end{bmatrix}$

now, RREF of $\left[\vec{c}_1 \ \vec{c}_2 \ \vec{c}_5 \middle| \vec{s}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$

which means $\boxed{5\vec{c}_1 - 3\vec{c}_2 + \vec{c}_5 = \vec{s}}$

NOTE: if you add the last three cols. of RREF C you get $\begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ this is NOT $\vec{s}$.

But it IS $5\vec{K}_1 - 3\vec{K}_2 + \vec{K}_5$ where the $\vec{K}_i$'s are the cols. of RREF C.

Know WHY these weights are the same in these two LC's!