

1. The consumption matrix  $C$  for an economy with three sectors  $G$ ,  $H$  and  $M$  and the final demand vector  $\mathbf{d}$  of the open sector are  $C = \begin{bmatrix} 0.02 & 0.1 & 0.01 \\ 0.01 & 0.2 & 0.05 \\ 0.03 & 0.4 & 0.07 \end{bmatrix}$  and  $\mathbf{d} = \begin{bmatrix} 400 \\ 500 \\ 600 \end{bmatrix}$ , respectively.

1A) Find  $\mathbf{x}$ , the vector showing the total numbers of units of goods produced by the three sectors  $G$ ,  $H$  and  $M$ . Show all your work and your answer rounded to *TWO* digits after the decimal point.

1B) Each unit produced by  $H$  requires how many units of  $G$ 's product?

1C) Of the total number of units produced by  $M$ , how many are consumed by  $H$ ?

2. Let  $C = \begin{bmatrix} 2 & 1 & 3 & 4 & 1 \\ 4 & 3 & 5 & 6 & 7 \\ -8 & -1 & -15 & -22 & 14 \end{bmatrix}$ , then the RREF of  $C$  is  $\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ .

Label the columns of  $C$  as  $\mathbf{c}_1, \mathbf{c}_2, \dots$ .

2A) Find a basis for  $\text{Col}(C)$ . Don't write the vectors out; use the names  $\mathbf{c}_1$ , etc.

2B) Find the sum  $\mathbf{s}$  of the last three column vectors of  $C$ . Now,  $\mathbf{s}$  must be in  $\text{Col}(C)$ . Indeed, express  $\mathbf{s}$  as a LC of the basis vectors from part 2A. Show any matrices (augmented, RREF'd, etc) involved in your work.