

Read directions carefully and *show your work*. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers.

1. (3 pts) Find $f'(t)$ if $f(t) = t^2 \cdot \ln(\sin t)$.

Use product & chain rule

$$f'(t) = 2t \ln(\sin t) + t^2 \cdot \frac{1}{\sin t} \cdot \cos t$$

$$= 2t \ln(\sin t) + \frac{t^2 \cos t}{\sin t}$$

2. (3 pts) Consider $y(x) = \sqrt{1 - \sqrt{x}}$, find $\frac{dy}{dx}$. Express your answer without using negative exponents.

Use chain rule

$$y(x) = \sqrt{1 - \sqrt{x}} = (1 - x^{1/2})^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} (1 - x^{1/2})^{-1/2} \cdot \frac{d}{dx} (1 - x^{1/2})$$

$$= \frac{1}{2\sqrt{1 - \sqrt{x}}} \cdot (-\frac{1}{2} x^{-1/2})$$

$$= \frac{1}{2\sqrt{1 - \sqrt{x}}} \cdot \frac{-1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-1}{4\sqrt{x}\sqrt{1 - \sqrt{x}}}$$

3. (4 pts) Find $\frac{dy}{dx}$ when $\sin y + x^2 y + y^3 = x^2$.

Use implicit differentiation

$$\cos y \cdot \frac{dy}{dx} + 2xy + x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 2x$$

$$\cos y \frac{dy}{dx} + x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 2x - 2xy$$

$$\frac{dy}{dx} (\cos y + x^2 + 3y^2) = 2x(1 - y)$$

$$\frac{dy}{dx} = \frac{2x(1 - y)}{\cos y + x^2 + 3y^2}$$