1. Find the derivative of each of the following functions. You do not need to simplify your answers.

1A. $m(x) = e^{\sin(2x)}$

$$m'(x) = \left(e^{\sin 2x}\right)\left(\sin(2x)\right)'$$
$$= \left(e^{\sin 2x}\right)\left((\cos(2x))(2x)'\right)$$
$$= \left(e^{\sin 2x}\right)\left(\cos(2x)\right)2$$

ALTERNATE NOTATION

$$m(x) = \exp(\sin(2x))$$
$$\therefore m'(x) = \exp(\sin(2x)) \cdot (\sin(2x))'$$
$$= \exp(\sin(2x)) \cdot \cos(2x) \cdot (2x)'$$
$$= \exp(\sin(2x)) \cdot \cos(2x) \cdot 2$$

1B. $q(x) = (x^3 + \log_2 x)\, x^5$

PRODUCT rule:

$$q'(x) = \left(3x^2 + \frac{1}{\ln 2 \cdot x}\right)\left(x^5\right) + \left(x^3 + \log_2 x\right)\left(5x^4\right)$$

1C. $k(x) = \ln(\sqrt[3]{x^4 + \cos(7x)} + x^5)$ CHAIN RULE (3 times!)

$$k'(x) = \frac{1}{(x^4+\cos(7x))^{1/3} + x^5} \cdot \left((x^4+\cos(7x))^{1/3} + x^5\right)'$$
$$= \quad '' \quad \cdot \left(\tfrac{1}{3}(x^4+\cos(7x))^{-2/3} \cdot (x^4+\cos(7x))' + 5x^4\right)$$

← CHAIN RULE ↓

$$= \quad '' \quad \cdot \left(\quad '' \quad \cdot (4x^3 - \sin(7x)(7x)') + 5x^4\right)$$
$$= \frac{1}{(x^4+\cos(7x))^{1/3} + x^5} \cdot \left(\tfrac{1}{3}(x^4+\cos(7x))^{-2/3}(4x^3 - \sin(7x)\cdot 7) + 5x^4\right)$$

1D. $p(x) = \dfrac{\sin(x) + x^6}{x^5 + \cos(x)}$

$$p'(x) = \frac{(x^5+\cos(x))(\cos(x) + 6x^5) - (5x^4 - \sin x)(\sin(x) + x^6)}{(x^5+\cos(x))^2}$$

THIS QUIZ CONTINUES ON THE OTHER SIDE!

2. The equation $x^4 + y^3 = x^2y^2 + 1$ implicitly defines y as a function of x, and a graph of this equation is shown at the bottom of the page.

2A. Use implicit differentiation to find y'.

Differentiate both sides of the equation to get:

$$\frac{d}{dx}\left(x^4+y^3\right) = \frac{d}{dx}\left(x^2y^2+1\right)$$

$$4x^3 + 3y^2y' = \underbrace{2xy^2 + x^2 2y\cdot y'}_{\text{(product rule on } x^2y^2)} + 0$$

solve for y':

$$3y^2y' - x^2 2yy' = 2xy^2 - 4x^3$$

$$y'(3y^2 - x^2 2y) = 2xy^2 - 4x^3 \; ; \; y' = \frac{2xy^2 - 4x^3}{3y^2 - x^2 2y}$$

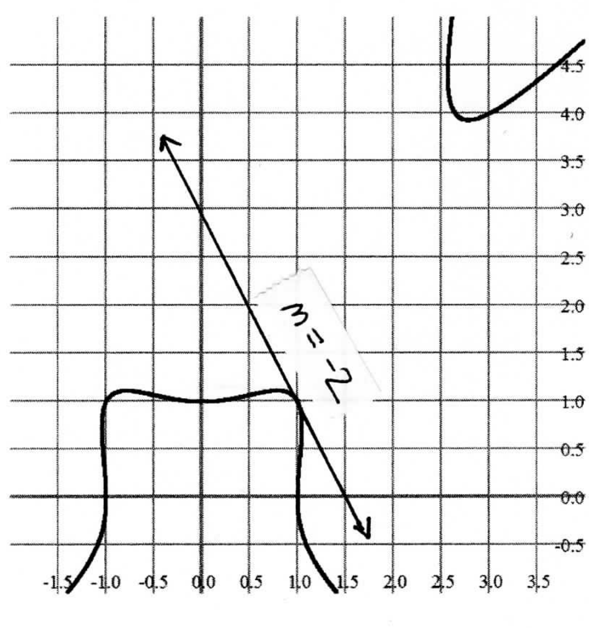
2B. The graph implies that $(1, 1)$ is a solution of the equation; show that $(1, 1)$ does indeed satisfy the equation.

does $1^4 + 1^3 = 1^2\cdot 1^2 + 1$? does $1+1 = 1\cdot 1 + 1$? does $2 = 2$? yes to all.

2C. Use the answer to 2A to find the slope of the graph of the equation at $(1, 1)$.

$$\text{so } y'\Big|_{(x,y)=(1,1)} = \frac{2\cdot 1\cdot 1^2 - 4\cdot 1^3}{3\cdot 1^2 - 1^2\cdot 2} = \frac{2-4}{3-2} = \frac{-2}{1} = -2$$

this seems reasonable; i've sketched the tangent line here



Q5 v2 p2