1. Find the derivative of each of the following functions. You do not need to simplify your answers.

1A. $p(x) = \dfrac{x^6 + \cos(x)}{\sin(x) + x^5}$

By the quotient rule, $p'(x) = \dfrac{(\sin(x) + x^5)\cdot(x^6 + \cos(x))' - (\sin(x) + x^5)'\cdot(x^6 + \cos(x))}{(\sin(x) + x^5)^2}$

$= \dfrac{(\sin(x) + x^5)(6x^5 - \sin(x)) - (\cos(x) + 5x^4)(x^6 + \cos(x))}{(\sin(x) + x^5)^2}$

1B. $m(x) = \sin(e^{2x})$

by the chain rule,

$m'(x) = \cos(e^{2x})\cdot(e^{2x})'$

but this part also requires the chain rule since $(2x)$ is the "inside function"; we get $(e^{2x})' = e^{2x}\cdot(2x)' = e^{2x}\cdot 2$

finally: $m'(x) = \cos(e^{2x})(e^{2x})(2)$

Alternate Notation:

$m(x) = \sin(\exp(2x))$

so

$m'(x) = \cos(\exp(2x))\cdot(\exp(2x))'$

$= \cos(\exp(2x))\cdot\exp(2x)(2x)'$

$= \cos(\exp(2x))\cdot\exp(2x)\cdot 2$

$= \cos(e^{2x})\cdot e^{2x}\cdot 2$

1C. $k(x) = \ln\left(x^2 + \sqrt[3]{x^2 + \cos(7x)}\right)$

$k'(x) = \dfrac{1}{x^2 + (x^2 + \cos(7x))^{1/3}} \cdot \left[\left(x^2 + (x^2 + \cos(7x))^{1/3}\right)\right]'$

a chain rule right here

$= \quad " \quad \left[2x + \tfrac{1}{3}(x^2 + \cos(7x))^{-2/3}\cdot(x^2 + \cos(7x))'\right]$

$= \quad " \quad \left[\quad " \quad\quad " \quad \cdot (2x + (-\sin 7x)(7x)')\right]$

$= \dfrac{1}{x^2 + (x^2 + \cos(7x))^{1/3}}\left[2x + \tfrac{1}{3}(x^2 + \cos(7x))^{-2/3}\cdot(2x - \sin(7x)\cdot 7)\right]$

1D. $q(x) = x^4(x^5 + \log_2 x)$

$q'(x) = (4x^3)(x^5 + \log_2 x) + (x^4)\left(5x^4 + \frac{1}{\ln 2}\cdot\frac{1}{x}\right)$

THIS QUIZ CONTINUES ON THE OTHER SIDE!

2. The equation $x^3 + y^4 = x^2y^2 + 1$ implicitly defines y as a function of x, and a graph of this equation is shown at the bottom of the page.

2A. Use implicit differentiation to find y'.

Differentiate both sides with respect to x gives :

$$3x^2 + 4y^3 \cdot y' = \underbrace{2x\,y^2 + x^2\,2yy'}_{\text{product rule applied to } x^2 \cdot y^2} + 0$$

$$4y^3y' - x^2 2yy' = 2xy^2 - 3x^2$$

$$y'(4y^3 - x^2 2y) = 2xy^2 - 3x^2$$

$$y' = \frac{2xy^2 - 3x^2}{4y^3 - x^2 2y}$$

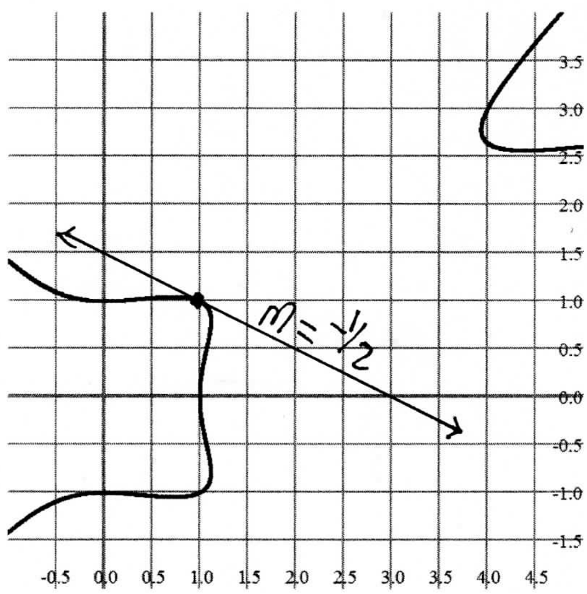
2B. The graph implies that $(1, 1)$ is a solution of the equation; show that $(1, 1)$ does indeed satisfy the equation.

does $1^3 + 1^4 = 1^2 \cdot 1^2 + 1$? does $1+1 = 1 \cdot 1 + 1$?

does $2 = 1 + 1$? yes.

2C. Use the answer to 2A to find the slope of the graph of the equation at $(1, 1)$.

$$\text{So } y'\Big|_{(x,y)=(1,1)} = \frac{2 \cdot 1 \cdot 1^2 - 3 \cdot 1^2}{4 \cdot 1^3 - 1^2\, 2 \cdot 1} = \frac{2-3}{4-2} = \frac{-1}{2}$$



(this seems reasonable if you sketch the tangent line here, as I've done)

Q5 V1 p2