

Math 205 Quiz 5

Name:

1. What are the three requirements for a subset H of a vector space V to be a subspace?

- (a) $\vec{0}$ must be in H .
- (b) H is closed under addition. For any \vec{x}, \vec{y} in H , $\vec{x} + \vec{y}$ must be in H too.
- (c) H is closed under scalar multiplication. For any \vec{x} in H and any scalar c , $c\vec{x}$ must be in H too.

2. Given $B = \begin{bmatrix} 3 & 2 & 2 & 3 \\ 0 & -2 & 0 & 8 \\ 12 & 1 & 8 & 4 \end{bmatrix}$. Consider the following QUICK questions.

(a) Is $\vec{v} = \begin{pmatrix} -2 \\ 0 \\ 3 \\ 0 \end{pmatrix}$ in $\text{Nul}(B)$? Why or why not?

$B(\vec{v}) = \vec{0}$. So, by definition \vec{v} is in $\text{Nul}(B)$.

(b) Find a vector in $\text{Col}(B)$.

$$\begin{bmatrix} 3 \\ 0 \\ 12 \end{bmatrix}$$

(c) Find a vector in $\text{Row}(B)$.

$$[3 \quad 2 \quad 2 \quad 3] \text{ or } \begin{bmatrix} 3 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$

3. Find a basis of the $\text{Nul}(A)$ and $\text{Col}(A)$.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & 2 & 1 \\ 2 & 0 & -5 \\ 3 & 1 & -1 \end{bmatrix} \text{ and } rref(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis for } \text{Col}(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -5 \\ -1 \end{bmatrix} \right\}, \text{Nul}(A) = \vec{0}. \text{ Either basis } = \{\} \text{ or basis } = \{\vec{0}\}.$$

4. Short Answer

- (a) How many vectors are in a basis for \mathbb{R}^3 ? 3
- (b) How many vectors are in a basis for \mathbb{P}_3 ? 4
- (c) How many vectors are in a basis for $M_{3 \times 3}(\mathbb{R})$? 9