Name: Solutions

Math 105: Fall 2012
Quiz 3: October 15

Correct answers accompanied by incorrect or incomplete work will not receive full credit. Good Luck!

1. Find the derivatives of the following functions.

   (a) $f(x) = 7^x - e^x + \ln x + e^\pi$

   $f'(x) = (\ln 7) 7^x - e^x + \frac{1}{x} + 0$

   (b) $f(x) = 6 \sin x + \frac{1}{2} \cos x$

   $f'(x) = 6 \cos x - \frac{1}{2} \sin x$

2. Find the solution to the IVP: $y' = 7e^t - 8 \sin t$, $y(0) = 5$.

find antiderivative of $y' = 7e^t - 8 \sin t$

$y = 7e^t + 8 \cos t + C$

(check $y' = 7e^t - 8 \sin t + 0$ ✓)

solve for C

$5 = y(0) = 7e^0 + 8 \cos(0) + C$

$5 = 7 + 8 + C$

$-10 = C$

so sol'n to IVP is

$\boxed{y = 7e^t + 8 \cos t - 10}$

3. Let $f(x) = \frac{x^4}{4} - \frac{2x^3}{3} + 12.$

(a) Prove that $x_1 = 0$ and $x_2 = 2$ are stationary points on f.

$f'(x) = x^3 - 2x^2$

$f'(0) = 0^3 - 2(0)^2 = 0$ and $f'(2) = 2^3 - 2(2^2) = 8 - 8 = 0$

Since $f'(0) = 0$ and $f'(2) = 0$ $x_1 = 0$ and $x_2 = 2$ are stationary points

(b) In the following chart, circle the correct response (positive or negative). Justify your answers without drawing any graphs.

x	$(-\infty, 0)$	0	$(0, 2)$	2	$(2, \infty)$
$f'(x)$	on the interval $(-\infty, 0)$, $f'(x) =$ positive/negative		on the interval $(0, 2)$, $f'(x) =$ positive/negative		on the interval $(2, \infty)$, $f'(x) =$ positive/negative
	$f'(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$		$f'(1) = (1)^3 - 2(1)^2 = 1 - 2 = -1$		$f'(3) = (3)^3 - 2(3^2) = 27 - 18 = 9$

(c) At $x = 0$, does $f(x)$ have a local maximum, local minimum, or neither? Use the chart in (b) to justify your answer.

neither b/c at local max/min the first derivative changes sign, but f' doesn't change sign at $x = 0$ as indicated by negative / negative on either side of 0 in chart above.

(d) At $x = 2$, does $f(x)$ have a local maximum, local minimum, or neither? Use the chart in (b) to justify your answer.

local min b/c the graph of f is decreasing to the left of 2 (as indicated by $f'(x) = $ neg in the interval $(0, 2)$.) and f is increasing to the right of 2 (as indicated by $f'(x) = $ pos on the interval $(2, \infty)$).