

1. Suppose  $M = \begin{bmatrix} 5 & 3 & 1 \\ 20 & 10 & b \end{bmatrix}$ ,  $N = \begin{bmatrix} c & s & 2 \\ c & t & 5 \\ c & 3 & 3 \end{bmatrix}$  and  $MN = \begin{bmatrix} 36 & 36 & a \\ z & 280 & 240 \end{bmatrix}$ .

Find the values of all those unknowns. Hint: it will be easy if you find them in more-or-less alphabetical order. There are many equations you can write down, but aim for those with as few unknowns as possible.

$$a = 5 \cdot 2 + 3 \cdot 5 + 1 \cdot 3 = 10 + 15 + 3 = 28 \quad (a \text{ is } 1^{\text{st}} \text{ row} \times 3^{\text{rd}} \text{ column})$$

$$20 \cdot 2 + 10 \cdot 5 + b \cdot 3 = 240$$

$$(240 \text{ is } 2^{\text{nd}} \text{ row} \times 3^{\text{rd}} \text{ column})$$

$$40 + 50 + 3b = 240$$

$$3b = 240 - 90 = 150$$

$$b = 50$$

$$5c + 3c + 1c = 36$$

(36 in the 1<sup>st</sup> row, 1<sup>st</sup> column of MN)  
using the

$$9c = 36 \quad c = 4$$

Answers:  $a = \boxed{28}$     $b = \boxed{50}$     $c = \boxed{4}$

$$\begin{cases} 5s + 3t + 3 = 36 \\ 20s + 10t + 3b = 280 ; 3b = 150 \text{ so} \end{cases}$$

$$\begin{cases} 5s + 3t = 33 \\ 20s + 10t = 130 \end{cases} \leftrightarrow \left[ \begin{array}{cc|c} 5 & 3 & 33 \\ 20 & 10 & 130 \end{array} \right] \sim \left[ \begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 1 \end{array} \right] \Rightarrow \begin{cases} s = 6 \\ t = 1 \end{cases}$$

$$z = \text{"2<sup>nd</sup> row} \times \text{1<sup>st</sup> col"} = 20c + 10c + bc = (20 + 10 + 50)(4)$$

$$s = \boxed{6} \quad t = \boxed{1} \quad z = \boxed{320} = 80 \cdot 4 = 320$$

BONUS: How many pairs of numbers have to be multiplied to find the product of a  $3 \times 7$  by a  $7 \times 10$  matrix? Explain. (No credit for a guess).

the product has  $3 \times 10 = 30$  entries, each of which requires 7 pairs of numbers to be multiplied  $\therefore$  210 total multiplications