

Name: \_\_\_\_\_

---

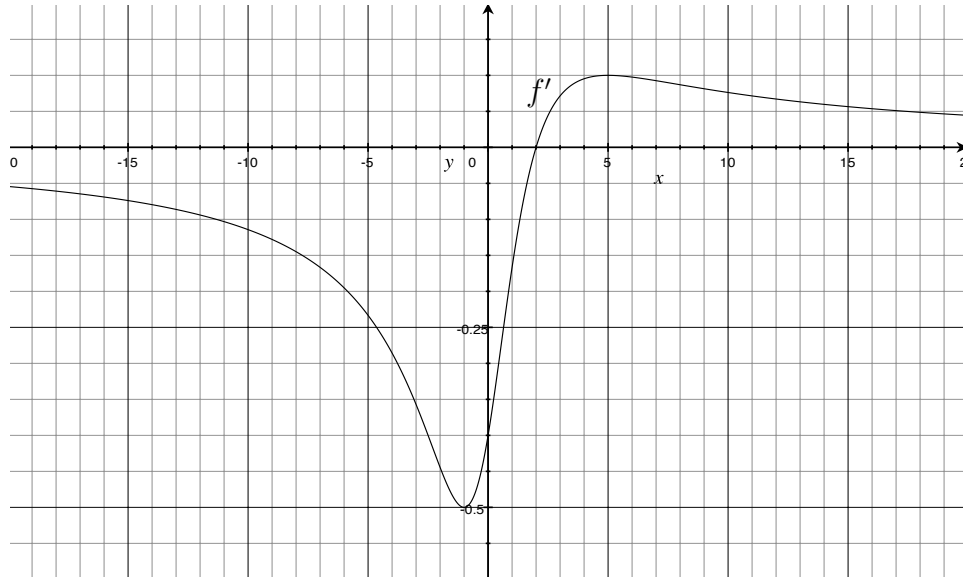
1. **Do not open this booklet until you are told to do so.**
  2. Try not to separate the pages. If they do become separated, write your names on every page and point this out to your proctor when you hand it in.
  3. Show an appropriate amount of work (including appropriate explanation) for each problem and not just the final answer. Include units in your answer where that is appropriate.
  4. You may use any calculator functionally equivalent to a TI-83/TI-83+ or TI-84/TI-84+. Use of calculators with more functionality than these is not allowed.
  5. **Turn off all cell phones and pagers, and remove all headphones.**
- 

**Proficiency Level on Chapter 1:**\_\_\_\_\_

**Proficiency Level on Chapter 2:**\_\_\_\_\_

### Chapter 1 Proficiency Test - Problem 1

The derivative,  $f'(x)$  of a function is shown in the graph below. In the problems which follow

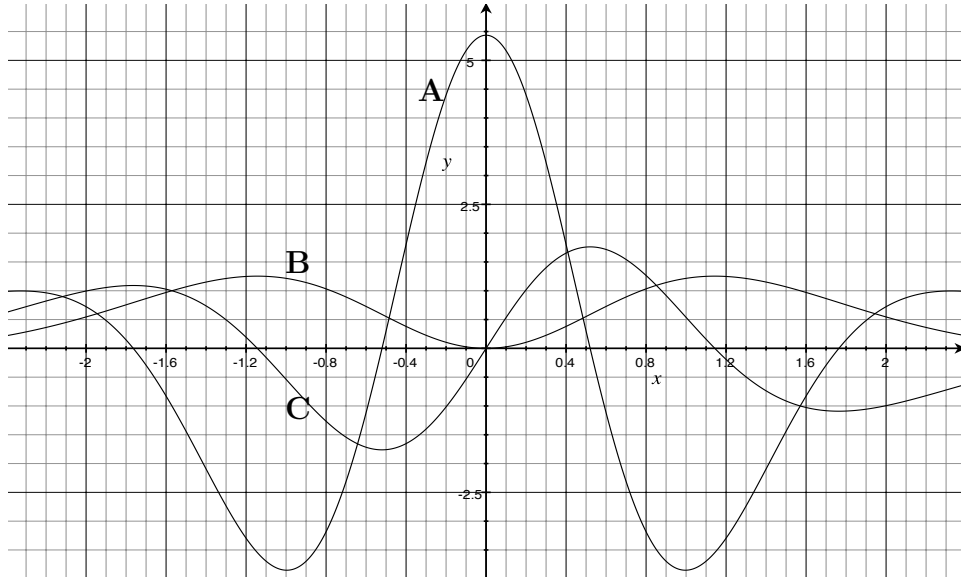


please **explain your answers**.

- (a) On which intervals, if any, is  $f$  increasing?
  
- (b) At which values of  $x$ , if any, does  $f$  have a local extrema? If there is such a point, is it a local maximum or a local minimum point?
  
- (c) On which intervals, if any, is  $f$  concave down?
  
- (d) On which intervals, if any, is  $f''$  positive?
  
- (e) At which values of  $x$  does  $f$  change concavity?
  
- (f) Sketch a graph of  $f$  on the axis given above.

### Chapter 1 Proficiency Test - Problem 2

The figure shows the graph of  $f$ ,  $f'$ , and  $f''$ . Identify each curve, and explain your choices.



## Chapter 2 Proficiency Test - Problem 1

(I) Suppose that  $f(x)$  and  $g(x)$  are defined for all  $x$  and that

$$\lim_{x \rightarrow 0} f(x) = \frac{1}{2} \quad \text{and} \quad \lim_{x \rightarrow 0} g(x) = \sqrt{2}.$$

Find the limits as  $x \rightarrow 0$  of the following functions.

(a)  $\lim_{x \rightarrow 0} [-g(x)] =$

(b)  $\lim_{x \rightarrow 0} [g(x) \cdot f(x)] =$

(c)  $\lim_{x \rightarrow 0} \left[ \frac{1}{f(x)} \right] =$

(d)  $\lim_{x \rightarrow 0} \left[ \frac{f(x) \cdot \cos(x)}{x - 1} \right] =$

(II) It can be shown that the inequalities

$$1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2 \cos x} < 1$$

hold for all values of  $x$  close to zero. What, if anything, does this tell us about

$$\lim_{x \rightarrow 0} \frac{x \sin x}{2 - 2 \cos x}.$$

(III) Find

$$\lim_{x \rightarrow 1^-} \frac{x - 1}{|x - 1|}, \quad \lim_{x \rightarrow 1^+} \frac{x - 1}{|x - 1|}, \quad \text{and} \quad \lim_{x \rightarrow 1} \frac{x - 1}{|x - 1|}.$$

**Chapter 2 Proficiency Test - Problem 2**

(I) Use the limit definition of the derivative to find  $f'(x)$  given that  $f(x) = x^2 + x + 1$

(II) Find the equation of the tangent line to  $f(x)$  at  $x = 2$

(III) Find the antiderivative for the function

$$f(x) = 6x^5 + \frac{5}{x^{10}} + \frac{4}{\sqrt{x}} + \pi$$

**Chapter 2 Proficiency Test - Problem 3**

(I) Where does  $f(x) = 1 + 2x^3 - x^4$  have a local maxima or local minima and what is the function value at this point?

(II) What is the area of the largest rectangle whose base is on the x-axis and whose upper corners are on the curve  $y = 12 - x^2$ .