Name:_____

Exam 1- In-Class Portion

Show all your work to receive full credit for a problem.

1. (12 pts) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = -3\mathbf{j} + \mathbf{k}$.

   (a) Find a vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

   (b) Let $\mathbf{x}_0 = (1, 2, 1)$. Find the equation for the plane going through the point $\mathbf{x}_0$ and with normal vector the one you found in part (a).

   (c) Find a parametric equation for the plane in part (b).

2. (12 pts) Let $\mathbf{a} = (2, 1, 2)$.

   (a) What is the magnitude of $\mathbf{a}$?

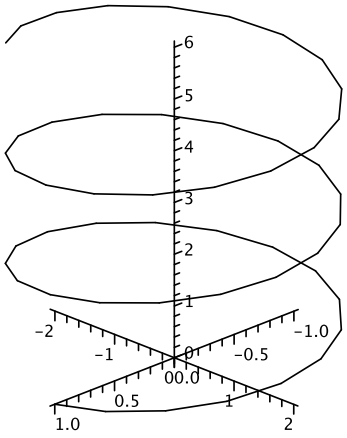
   (b) What is the midpoint between $\mathbf{a}$ and the origin?

   (c) Write the equation for the sphere centered at the origin and containing $\mathbf{a}$.

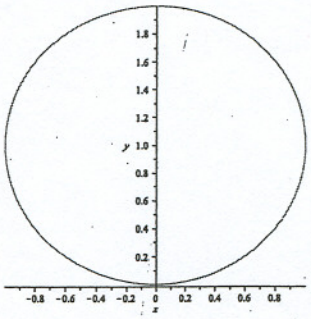
   (d) What would the equation you found in part (c) be if you were to write it in spherical coordinates?

3. (12 pts) Let $\mathbf{f}(t) = \cos(3t)\mathbf{i} + 2\sin(3t)\mathbf{j} + t\mathbf{k}$.

(a) Explain why it makes sense to say that $\mathbf{f}$ is a parametrization of the curve depicted below.



(b) Find the parametrization for the line tangent to the path described by $\mathbf{f}$ at $t = \frac{\pi}{6}$.

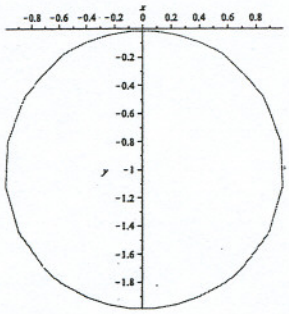


4. (12 pts) Let C be the circle of radius 1 and center (0,1), as depicted above. Match each of the following linear transformations to the image of C under that transformation. Explain each of your answers.

(a) $T(\mathbf{x}) = \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}$ X

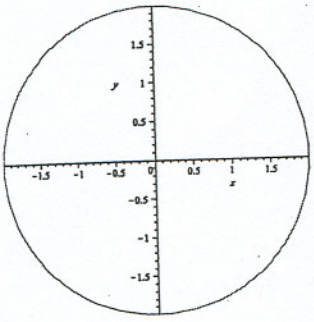
(b) $T(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ X

I

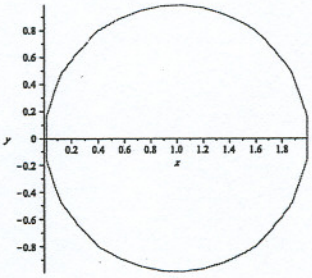


II

III



IV



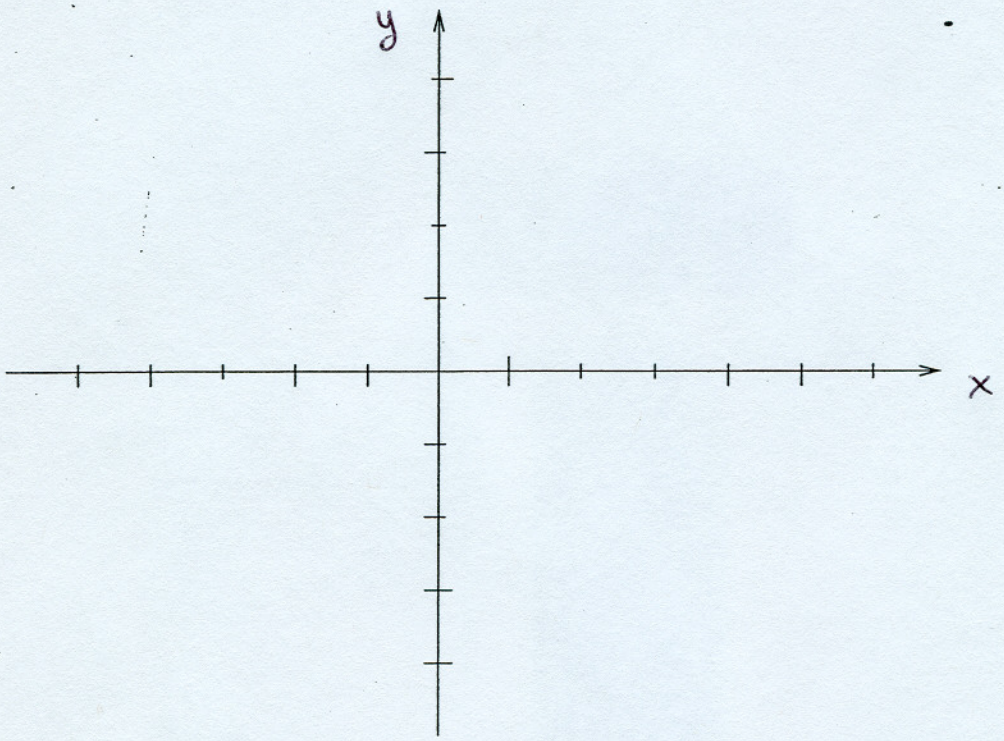
5. (12 pts) Let $p(x, y) = 3x^2 - 8xy + 6y^2$.

   (a) Explain why p is a quadratic form.

   (b) Find a symmetric matrix S such that $p(\mathbf{x}) = \mathbf{x}^T S\mathbf{x}$.

   (c) Use Sylvester's Theorem to show that this quadratic form is positive definite. What does this tell you about the graph of $p(x, y)$?

6. (6 pts) Let $\mathbf{f}$ denote the vector field defined by $\mathbf{f}(x, y) = (2y, -x)$. Sketch six values of the vector field in the coordinate system provided.



7. (6 pts) Let $g(x, y) = \sqrt{x^2 + y^2}$. Sketch the level curves for g, when $c = -1, 0, 1, 2$, in the coordinate axes below.