1. Let $\mathbf{a} = \overrightarrow{(2,\ 5,\ 7)}$ and $\mathbf{b} = \overrightarrow{(p,\ 4,\ 1)}$; suppose $\mathbf{a} \times \mathbf{b}$ is $\overrightarrow{(-23,\ 33,\ -17)}$. Find p. Show your work.

2. Let $\overrightarrow{\ell}_1(t)$ be the line parameterized by $\overrightarrow{(2,-5,1)} + t\overrightarrow{(3,-2,5)}$.
Let the parameteric equations for line $\overrightarrow{\ell}_2$ be $x = 4s + 5$, $y = 7s - 36$, and $z = 2s + 20$.

2A. It's a fact that these two lines intersect. Find t and s for which $\overrightarrow{\ell}_1(t) = \overrightarrow{\ell}_2(s)$. Show your work.

2B. What is the point of intersection?

2C. In the form $\overrightarrow{(x - x_0,\ y - y_0,\ z - z_0)} \cdot \mathbf{n} = 0$, what is an equation for the plane containing these two lines?

3. Let $\mathbf{a} = \overrightarrow{\langle 3, -2, 5\rangle}$ and $\mathbf{b} = \overrightarrow{\langle 4, 7, 2\rangle}$.

3a. What is the area of the parallelogram which has vectors **a** and **b** as two of its sides?

3b. Find the projection vector $\text{proj}_{\mathbf{b}}(\mathbf{a})$ of **a** onto **b**.

3c. Find a unit vector perpendicular to both **a** and **b**.

3d. To the nearest 1/10 of a degree, find in degrees the angle θ between **a** and **b**.

3e. Let P be the point at the tip of the vector **b**. What are the *cylindrical* coordinates of P?

4. Let $\mathbf{f}$ and $\mathbf{g}$ be differentiable vector-valued functions from $\mathbf{R}$ to $\mathbf{R}^3$.

What does theorem 1.10.3 tell us that $(\mathbf{f} \times \mathbf{g})'$ is in terms of $\mathbf{f}$, $\mathbf{f}'$, $\mathbf{g}$ and $\mathbf{g}'$?

5. Let $\mathbf{f}(t) = \overrightarrow{(t, t^2, t^3)}$, $\mathbf{g}(t) = \overrightarrow{(t^4, e^t, 0)}$ and $h(t) = \cos t$.

5a. Find and simplify both $(\mathbf{f} \times \mathbf{g})(t)$ and $(\mathbf{f} \times \mathbf{g})'$ for this specific pair of vector-valued functions.

5b. Find explicit formulas for both $\mathbf{f}(h(t))$ and $\mathbf{f}(h(t))'$.

6. Consider the sphere centered at $(3, -2, 5)$ with radius 7.

6a. What is the equation of this sphere?

6b. What is the equation of the plane tangent to this sphere at its "*south* pole"?

7. Consider an object moving along a path with its position parameterized by $\mathbf{g}(t) = \overrightarrow{(g_1(t), g_2(t))}$ where

$$g_1(t) = t^4 - \frac{4}{3}t^3 - 12t^2 + 40 \qquad \text{and} \qquad g_2(t) = t^3 - 3t.$$

(Assume t is in seconds and the units on the xy plane are in feet).

7a. Use your calculator to draw a decent, labeled sketch of this path for $t \in [-2.6,\ 3.2]$. Set your window to $[-30,\ 50] \times [-12,\ 25]$ and set the scales on the x and y axes to 10 and 10.

7b. What are all the times that the horizontal components of the velocity are 0?

7c. Find a parameterization for the line tangent to the path at $t = 3$.

7d. If the object leaves the path at $t = 3$ and follows the tangent line with the velocity it has at $t = 3$, where is it at $t = 7$?

7e. *Bonus!* this is a continuation of the problem on the previous page: To the nearest 1/10, when is the object moving slowest, and where is it? Explain!

8. Find the equation of the elliptic paraboloid given here: In addition to the two points labeled, it's tangent to the xy plane. Just FYI, the "contour curves" are at heights $z = 2, 4, 6, 8 \ldots$.