

**MATH 205A,B - LINEAR ALGEBRA
FALL 2015**

QUIZ 4

NAME: _____ **Section:**(Circle one) A(8 : 00) B(9 : 30)

Show ALL your work CAREFULLY.

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by

$$T(x_1, x_2) = (3x_1 - x_2, 2x_1 - x_2).$$

(a) Find all vectors \vec{x} such that $T(\vec{x}) = \vec{0}$.

If $T(\vec{x}) = \vec{0}$, then $3x_1 = x_2$ and $2x_1 = x_2$. It follows that $3x_1 = 2x_1$ or $x_1 = 0$ and thus $x_2 = 0$.

(b) Is T one to one? Explain.

From (b), $T(\vec{x}) = \vec{0}$ implies that $\vec{x} = \vec{0}$ and thus T is one to one.

(c) Find the matrix A of T so that $T(\vec{x}) = A\vec{x}$.

The associated matrix of T is given by

$$A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}.$$

(d) Is the matrix A invertible? If so, find A^{-1} .

To find A^{-1} in case A is invertible, we consider the augmented matrix

$$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 2 & -1 & 0 & 1 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & -1 & -2 & 3 \end{array} \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 2 & -3 \end{array} \right]$$

It follows that A is invertible and

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}.$$