Math 116 — First Midterm

October 5, 2012

Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 8 pages including this cover AND IS DOUBLE SIDED. There are 6 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	18	
2	20	
3	22	
4	14	
5	14	
6	12	
Total	100	

1. [18 points]

a. [6 points] Describe and sketch the surface defined by $z + 88 = -9x^2 + 54x - 4y^2 - 16y$.

b. [6 points] Write down the parametrization $r(t)$ for the intersection of this surface with the surface $z = x^2$.

c. [6 points] Calculate the equation for the tangent plane to the parabolic bowl at $(4, -2, 0)$.

2. [20 points]

a. [5 points] Check that the two lines with the following parametrizations intersect at the point $(1, 2, -4)$:

•$r_1(t) = [0, 2, -3] + t[1, 0, -1]$

•$r_2(s) = [-2, 3, -6] + s[3, -1, 2]$.

b. [7 points] Let v be the vector defined by $r_1(2) - r_1(1)$, and let w be the vector defined by $r_2(2) - r_2(1)$. Calculate the vector a defined as $a = (v \times w)$.

c. [5 points] Let $b = (x, y, z)$ be any vector such that b is perpendicular to a. Solve for the equation defining all such b.

d. [5 points] Graph the equation from part (c), and in the same graph, plot the curves r_1 and r_2.

3. [22 points] Given a point in rectilinear coordinates (x, y, z) there is a function $f(x, y, z) = (r, \theta, w)$ which gives us the cylindrical coordinates, and another function $g(x, y, z) = (\rho, \theta, \phi)$ which gives us the spherical coordinates. The Martians have a slightly different way of describing vectors (for a full description, read "The Martian Chronicles"). They use (a, b, c) which satisfy $(x, y, z) = M(a, b, c) = (3a\cos(b)\sin(c), 4a\sin(b)\sin(c), 7a\cos(c))$.

a. [8 points] Calculate the Jacobians of f and M.

b. [6 points] Plot the rectilinear coordinate $(1, 1, \sqrt{2})$ and convert to cylindrical and spherical coordinates.

c. [8 points] Sketch the Martian equation $a = 2$.

4. [14 points] Calculate the following limits, or demonstrate that they do not exist:

a. [7 points]

$$f(x,y) = \begin{cases} \frac{y^4}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

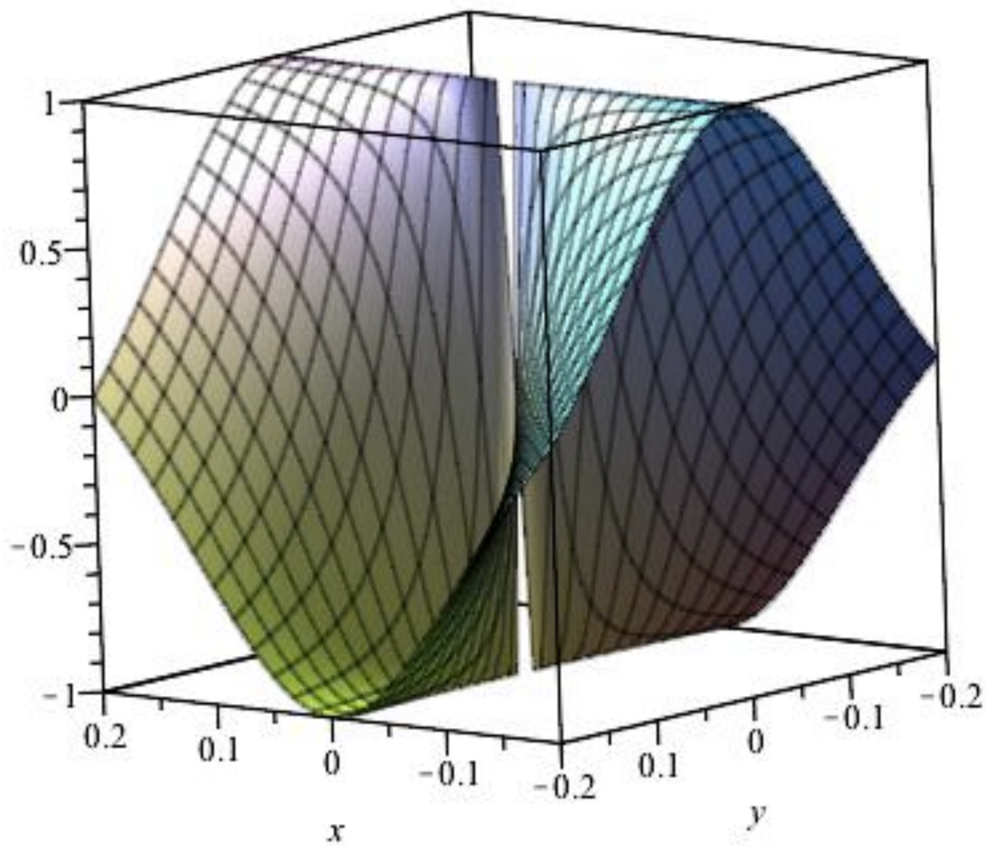
What is $\lim_{(x,y)\to(0,0)} f(x,y)$ or does it not exist?

b. [7 points] Does

$$\lim_{(x,y)\to(0,0)} \frac{\ln(1-x^2-y^2)+x^2+y^2}{x^2+y^2}$$

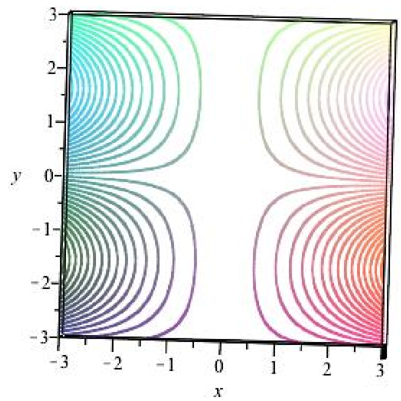
exist? If so, what is it?

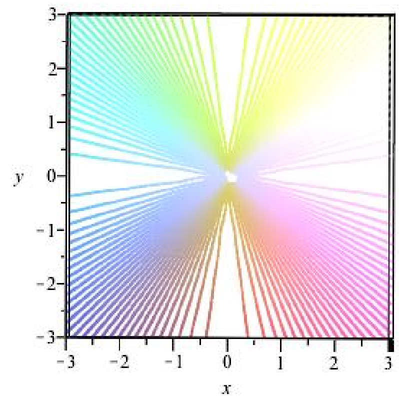
5. [14 points] Here is the graph of a function $f(x, y) = z$:

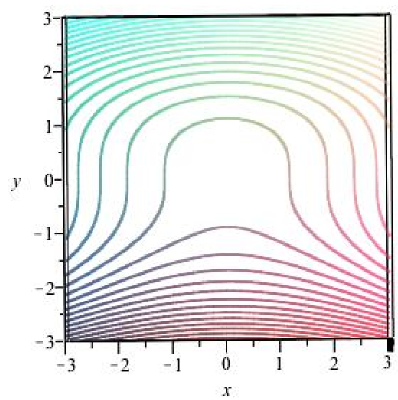


a. [7 points] What is the $\lim_{(x,y)\to(0,0)} f(x, y)$? Explain how you calculated it.

b. [7 points] Circle the plot of the level curves of $f(x, y)$. Briefly explain your choice.







6. [12 points]

a. [6 points] Sketch the image of the unit square under $f(x, y) = (x - y^2, y)$.

b. [6 points] Sketch the level curves for $f(x, y) = \frac{x^2}{4} + \frac{y^2}{9}$ with $c = 0, 1$.