Name: Solutions

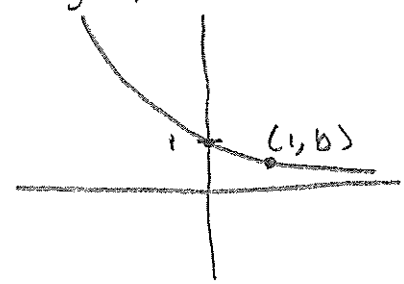
Math 105A: Fall 2012
Exam 1: October 5

**Correct answers accompanied by incorrect or incomplete work will not receive full credit.**

1. (8 points) Let $g(x) = b^x$ where $b$ is a constant and $0 < b < 1$.

   (a) What is the domain of $g(x)$?

   note graph is

   $(-\infty, \infty)$

   (b) Is 0 in the range of $g(x)$? Justify your answer.

   No b/c there is no value of x for which $b^x = 0$.

2. (4 points) Let $P(t)$ be the population of the US (in millions) $t$ years after 1800. What does the statement $P'(10) \approx 2.3$ mean in this context? Include units in your answer.

   In 1810 the population is growing at a rate of 2.3 million people per year

3. Suppose $g(x) = 3f(x) - 2$.

   (a) (5 points) Describe how the graphs of $g(x)$ and $f(x)$ are related. (Use words like horizontal, vertical, compressed, shifted, stretched, translated, etc.)

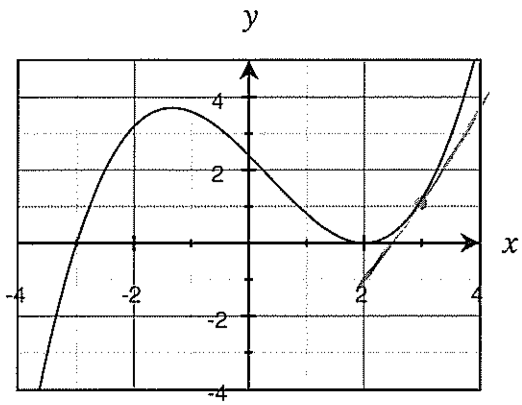
   first $f(x)$ is stretched vertically then shifted down 2 units to get $g(x)$.

   (b) (4 points) Let $f'(x) = (x^2 + 3)^{-1}$. Evaluate $g'(1)$.

   $g'(x) = 3f'(x) - 0 = 3f'(x)$

   $g'(1) = 3(1+3)^{-1} = \boxed{\frac{3}{4}}$

4. (5 points each) The graph below is a graph of $y = g''(x)$.




(a) Estimate $g'''(3)$. $=$ slope of tangent line to $g''(x)$ at $x=3$

$$g'''(3) \approx \frac{3.5-1}{4-3} = 2.5$$

(b) If possible, determine each of the following. Justify your answer.

i. The interval(s) for which $g$ is increasing.

To determine when $g$ is increasing we need to know when $g'$ is positive but this can't be determined from the $g''$ graph

ii. The interval(s) for which $g$ is concave up.

$g$ is concave up when $g''(x) > 0$

so $(-3,2) \cup (2,3.8)$

other acceptable answers

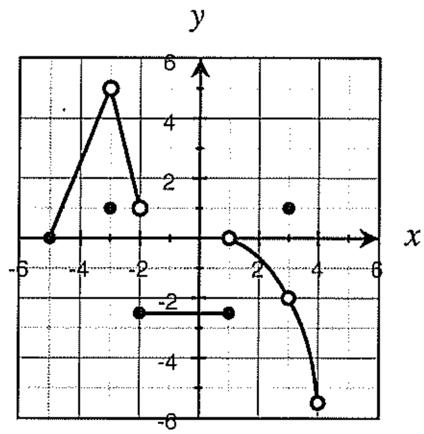
$(-3,2) \cup (2,4)$

$(-3,2) \cup (2,\infty)$

$(-3,4)$

$(-3,\infty)$

5. (4 points each) The graph of $f(x)$ is given. Solve the following (assume the tickmarks occur at 1, 2, etc).




(a) $\lim_{x \to 1^-} f(x)$ -2.5

(b) $\lim_{x \to 1^+} f(x)$ 0

(c) $f(1)$ -2.5

(d) $\lim_{x \to -2} f(x)$ DNE b/c left and right hand limits do not agree

(e) $\lim_{x \to 3} f(x)$ -2

(f) For what value(s) of $x$ is $f(x)$ NOT continuous?

x = -3, -2, 1, 3

6. (6 points) Let $f(x) = \sqrt{x+2}$. Use the limit definition of derivative to compute $f'(14)$.

$$f'(14) = \lim_{h \to 0} \frac{f(14+h) - f(14)}{h} = \lim_{h \to 0} \frac{\sqrt{14+h+2} - \sqrt{16}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{16+h} - 4}{h} \cdot \frac{(\sqrt{16+h} + 4)}{(\sqrt{16+h} + 4)} = \lim_{h \to 0} \frac{16 + h - 16}{h(\sqrt{16+h} + 4)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{16+h} + 4)} = \lim_{h \to 0} \frac{1}{\sqrt{16+h} + 4} = \frac{1}{\sqrt{16} + 4} = \boxed{\frac{1}{8}}$$

7. (6 points each) Let $f(x) = 3x^2 + \frac{\pi}{x^2} - \sqrt{x} + 12$. $= 3x^2 + \pi x^{-2} - x^{1/2} + 12$

(a) Find the derivative of $f$.

$$f' = 3(2)x^1 + \pi(-2)x^{-3} - \frac{1}{2}x^{-1/2} + 0$$

$$= 6x - \frac{2\pi}{x^3} - \frac{1}{2}x^{-1/2}$$

(b) Find an antiderivative of $f$.

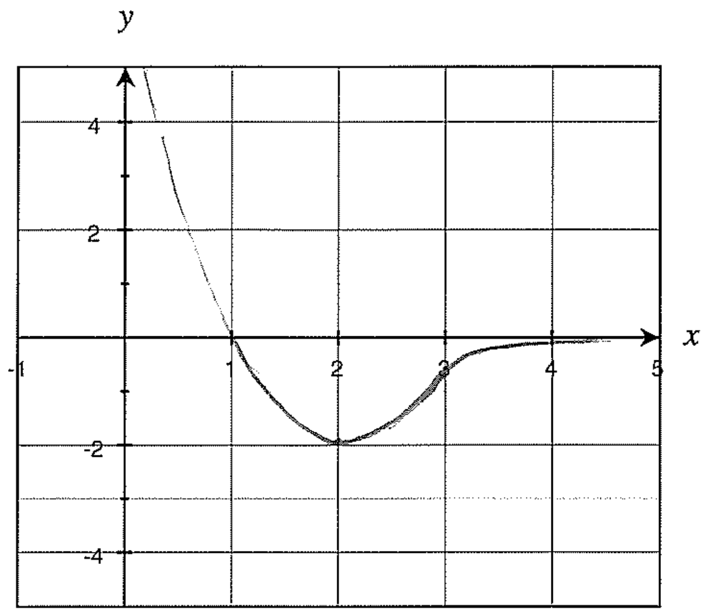
$$F = \frac{3x^3}{3} + \frac{\pi x^{-1}}{-1} - \frac{x^{3/2}}{3/2} + 12x + C$$

$$= 3x^3 - \frac{\pi}{x} - \frac{2}{3}x^{3/2} + 12x + C$$

8. (5 points each) Let $f(x)$ be a continuous function with the following properties:

- $f$ is positive on the interval $(-\infty, 1)$ and negative on the interval $(1, \infty)$.
- $f'(x) < 0$ on the interval $(-\infty, 2)$, $f'(2) = 0$, and $f'(x) > 0$ on the interval $(2, \infty)$.
- $f''(x) > 0$ on the interval $(-\infty, 3)$, $f''(3) = 0$ and $f''(x) < 0$ on the interval $(3, \infty)$.

(a) Sketch a possible graph of $f(x)$.




(b) Suppose $F$ is an antiderivative of $f$. At which value(s) of $x$ in the interval $[0, 5]$ does $F$ have a local minimum? Justify your answer.

None: b/c F has a local minimum at a point where $F' = f$ changes from negative valued to positive valued, but that never happens.

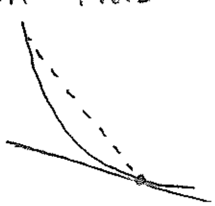
(c) Suppose $F$ is an antiderivative of $f$. Does $F$ have an inflection point? Justify your answer.

Yes, at $x=2$ b/c $F'' = f'$ and $f'$ changes sign at $x=2$

(d) Which value is larger: $\dfrac{f(0) - f(-3)}{3}$ or $f'(0)$? Justify your answer.

$\dfrac{f(0)-f(-3)}{3}$ is the slope of the secant line between the points $(0, f(0))$ and $(-3, f(-3))$.

On this region $f(x)$ is decreasing and concave up, like .



slope of dotted line $= \dfrac{f(0)-f(-3)}{3}$

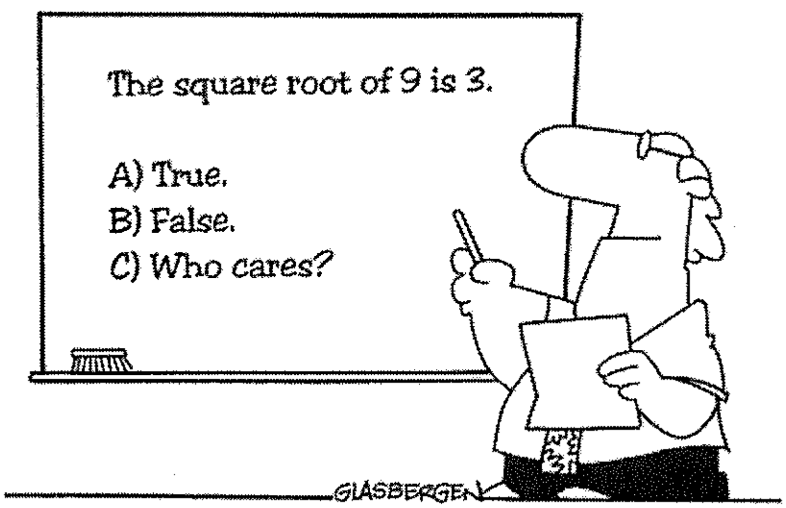
slope of solid line $= f'(0)$

Both are negative but $f'(0)$ is larger (closer to 0).

9. (2 points each) Who do you think will win the World Series?

- Atlanta Braves
- Baltimore Orioles
- Cincinnati Reds
- Detroit Tigers
- New York Yankees
- Oakland Athletics
- San Francisco Giants
- St. Louis Cardinals
- Texas Rangers
- Washington Nationals






**Many students actually look forward to Mr. Atwadder's math tests.**