TEST 1

Math 105
10/5/12

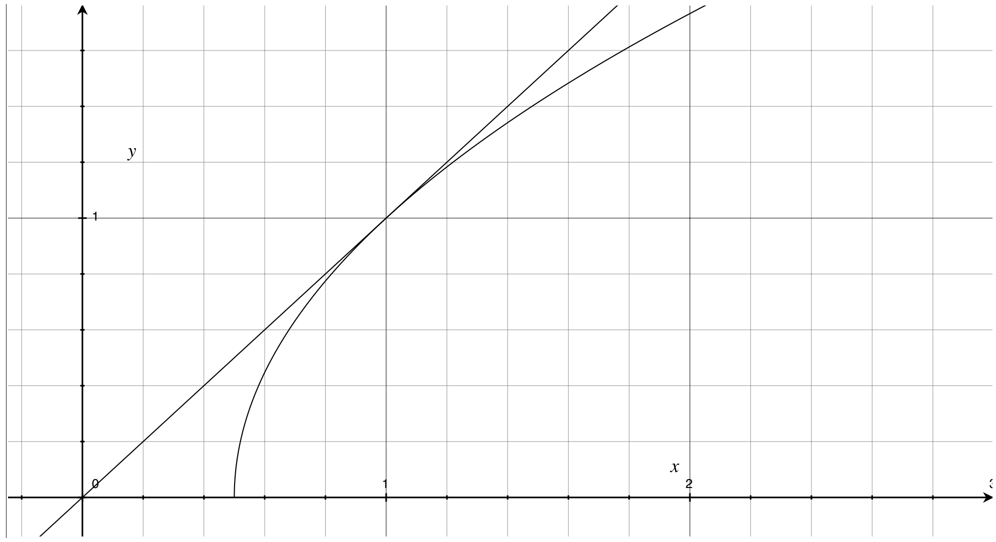
Name: ________________________
by writing my name I swear this work is my own

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements. Put a smiley face next to your name for one point.
- This test has 5 problems and is worth 92 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*16 points*)

a. (*4 pts*) Draw the tangent line to the below graph at $x = 1$. Estimate the slope of the tangent line at $x = 1$.



The slope is equal to 1. (.6,.6) and (1.6,1.6), slope is 1.

b. (*4 pts*) Using an appropriate **table of values**, find $\lim_{x\to 1} \frac{\sqrt{2x-1}-1}{x-1}$.

x	$\lim_{x\to 1} \frac{\sqrt{2x-1}-1}{x-1}$
.9	1.0557
.99	1.00505
1.001	.9995
1.01	.995

$\lim_{x\to 1} \frac{\sqrt{2x-1}-1}{x-1}$=1

c. (*8 pts*) Using the formal limit definition of the derivative ($\lim_{h\to 0}$, etc.), find $f'(1)$ for

$$f(x) = \sqrt{2x-1}$$

.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{\sqrt{2(1+h)-1}-1}{h} = \lim_{h\to 0} \frac{\sqrt{2(1+h)-1}-1}{h} \cdot \frac{\sqrt{2(1+h)-1}+1}{\sqrt{2(1+h)-1}+1}$$

$$= \lim_{h\to 0} \frac{(2(1+h)-1)-1}{h} = \lim_{h\to 0} \frac{2h}{h(\sqrt{2(1+h)-1}+1} = \lim_{h\to 0} \frac{2}{\sqrt{2(1+h)-1}+1} = \frac{2}{2} = 1$$

2. (*15 points*) The graph of $f(x)$ is given. Solving the following (assume the tickmarks occur at 1, 2, etc).

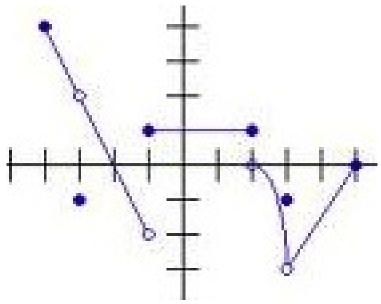
a. (*3 pts*) $\lim_{x\to -1^-} f(x) = -2$

b. (*3 pts*) $\lim_{x\to -1^+} f(x) = 1$

c. (*3 pts*) $f(-1) = 1$

d. (*3 pts*) $\lim_{x\to 2} f(x) = DNE$

e. (*3 pts*) $\lim_{x\to -3} f(x) = 2$



3. (*24 points*) Determine the following. For part **a.** rewrite the final answer to remove all negative and fractional exponents.

a. (*6 pts*) $\frac{d}{dx}\left(2\sqrt[4]{x^3} + \frac{1}{4x} - 2x^{-3} + 5x + 2\right)$

$$\frac{6}{4\sqrt[4]{x}} - \frac{1}{4x^2} + \frac{6}{x^4} + 5$$

b. (*7 pts*) Find $f'(x)$ and $f''(x)$ for $f(x) = ax^n + bx^{n-1} + c$.

$$f'(x) = anx^{n-1} + b(n-1)x^{n-2}$$

$$f''(x) = an(n-1)x^{n-2} + b(n-1)(n-2)x^{n-3}$$

c. (*5 pts*) Is $y = Cx^2$ a solution to the differential equation $x^2y'' - 2xy' + 2y = 0$ for any constant C? Justify your answer.

$$y = Cx^2, y' = 2Cx, y'' = 2C$$

$$x^2(2C) - 2x(2Cx) + 2(Cx^2) = 2Cx^2 - 4Cx^2 + 2Cx^2 = 0$$

Yes, it solves the DE.

d. (*6 pts*) Solve the following differential equation with initial condition $y(1) = 4$.

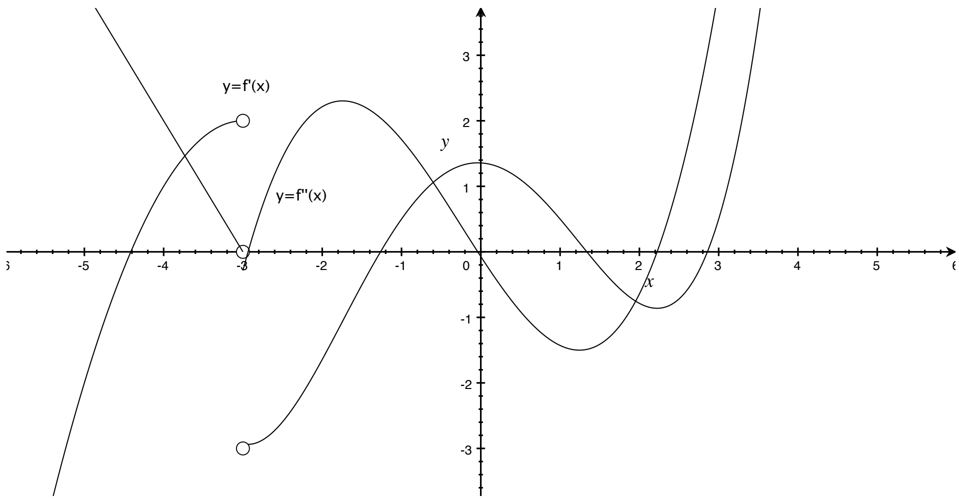
$$y' = 2x - x^{-2}$$

$$y = x^2 + x^{-1} + C$$

$y(1) = 1^2 + 1/1 + C = 4$ which implies $C = 2$

$$y = x^2 + x^{-1} + 2$$

4. (*28 points*) The following is a graph of $f'(x)$, NOT $f(x)$. $f(x)$ is a continuous function. Assume the graph continues off to negative and positive infinity.



a. (*4 pts*) On which intervals is $f(x)$ increasing or decreasing?

INC: (-4.4,-3) ∪ (-1.2,1.35) ∪ (2.8, ∞)
DEC: (∞,-4.4) ∪ (-3,1.2) ∪ (1.35,2.8)

b. (*4 pts*) On which intervals is $f''(x)$ positive or negative?

POS: $(-\infty, 0)$ ∪ (2.2,∞)
NEG: (0,2.2)

c. (*4 pts*) Draw and label the graph of the $f''(x)$ on the graph.

See above graph.

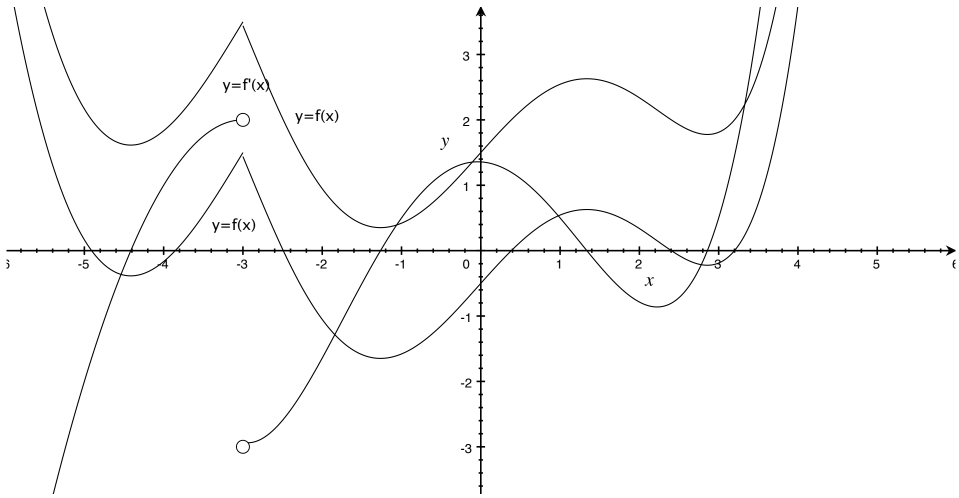
d. (*6 pts*) At what x values does $f(x)$ have local maximum or minimum? Identify which are maximums and which are minimums.
(Hint: Clearly $f(x)$ is not differentiable at $x = -3$. Since $f(x)$ is continuous that means at $x = -3$, $f(x)$ turns a sharp corner or comes to a point. We can't define the derivative at such a point (think about the graph of $|x|$). Is $x = -3$ a local max, min, or neither? Consider it with the others.]

x=-4.4, -1.2, 2.8 are min, $f''(x) > 0$ for these points were $f'(x) = 0$.
x=1.35 is a max, $f''(x) < 0$ and $f^prime(x) = 0$ at this x-value.
x=-3 is a max, $f'(x)$ is positive before and negative after x=-3.

e. (*4 pts*) At what x-values does $f(x)$ have inflection points?

x=0,2.2

f. (*6 pts*) Sketch and label **2** possible graphs of $f(x)$ on the graph below. Remember, $f(x)$ must be continuous.



5. (*8 points*) The cubic $f(x) = \dfrac{x^3}{3} - ax^2 + bx + c$ has an inflection point at $x = 1$. At $x = 3$ the tangent line to the graph is $y = 3x - 1$. Determine a, b, and c.

$$f'(x) = x^2 - 2ax + b$$

$$f''(x) = 2x - 2a$$

$x = 1$ is an inflection point implies that $f''(1) = 0$.

$$2(1) - 2a = 0 \rightarrow a = 1$$

At $x = 3$, the tangent line is $y = 3x - 1$. The slope of the tangent line is 3 when x=3. So, the derivative of the function at $x = 3$ is 3.

$$3 = (3^2) - 2(3) + b \rightarrow 3 = 9 - 6 + b \rightarrow b = 0$$

Finally, the tangent line touches the graph at the point (3,3*3-1) or (3,8). The point (3,8) is a point on the cubic.

$$8 = \frac{3^3}{3} - 3^2 + c \rightarrow 8 = 9 - 9 + c \rightarrow c = 8$$

The cubic is $f(x) = \frac{x^3}{3} - x^2 + 8$.