

# TEST 1

Math 105  
10/5/12

Name: \_\_\_\_\_

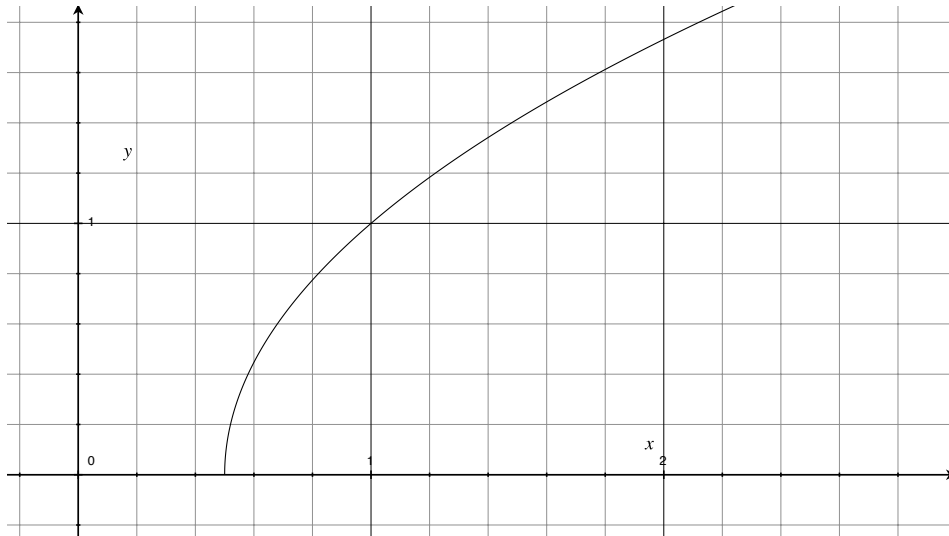
by writing my name I swear this work is my own

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements. Put a smiley face next to your name for one point.
- This test has 5 problems and is worth 100 points, It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (16 points)

a. (4 pts) Draw the tangent line to the below graph at  $x = 1$ . Estimate the slope of the tangent line at  $x = 1$ .



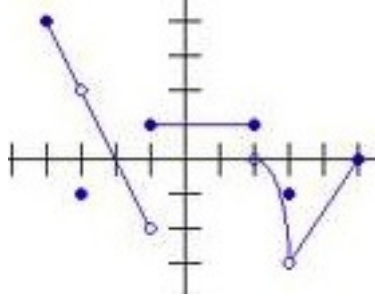
b. (4 pts) Using an appropriate **table of values**, find  $\lim_{x \rightarrow 1} \frac{\sqrt{2x-1} - 1}{x-1}$ .

c. (8 pts) Using the formal limit definition of the derivative ( $\lim_{h \rightarrow 0}$ , etc.), find  $f'(1)$  for

$$f(x) = \sqrt{2x-1}$$

**2.** (15 points) The graph of  $f(x)$  is given. Solving the following (assume the tickmarks occur at 1, 2, etc).

- a. (3 pts)  $\lim_{x \rightarrow -1^-} f(x) =$   
 b. (3 pts)  $\lim_{x \rightarrow -1^+} f(x) =$   
 c. (3 pts)  $f(-1) =$   
 d. (3 pts)  $\lim_{x \rightarrow 2} f(x) =$   
 e. (3 pts)  $\lim_{x \rightarrow -3} f(x) =$



**3.** (24 points) Determine the following. For part **a.** rewrite the function to remove all negative and fractional exponents.

a. (6 pts)  $\frac{d}{dx} \left( 2\sqrt[4]{x^3} + \frac{1}{4x} - 2x^{-3} + 5x + 2 \right)$

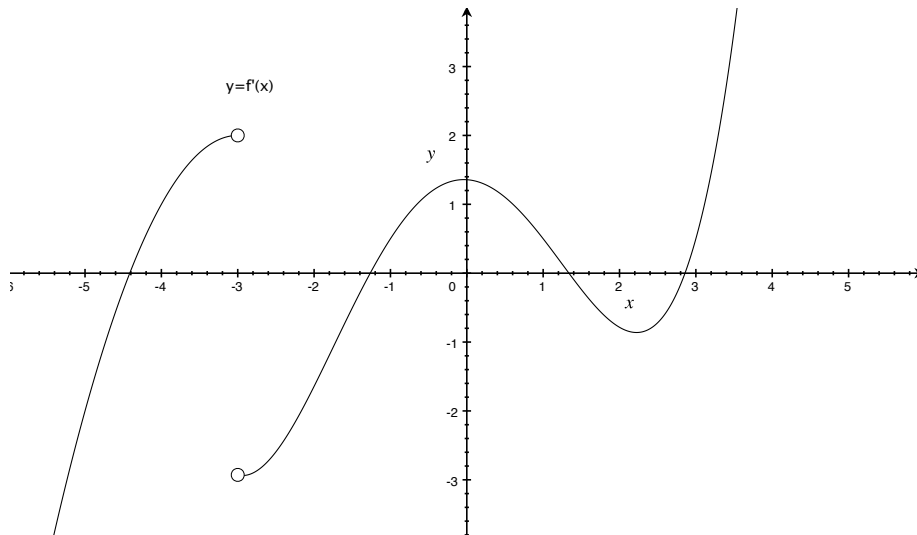
b. (7 pts) Find  $f'(x)$  and  $f''(x)$  for  $f(x) = ax^n + bx^{n-1} + c$ .

c. (5 pts) Is  $y = Cx^2$  a solution to the differential equation  $x^2y'' - 2xy' + 2y = 0$  for any constant  $C$ ? Justify your answer.

d. (6 pts) Solve the following differential equation with initial condition  $y(1) = 4$ .

$$y' = 2x - x^{-2}$$

4. (28 points) The following is a graph of  $f'(x)$ , NOT  $f(x)$ .  $f(x)$  is a continuous function. Assume the graph continues off to negative and positive infinity.



a. (4 pts) On which intervals is  $f(x)$  increasing or decreasing?

b. (4 pts) On which intervals is  $f''(x)$  positive or negative?

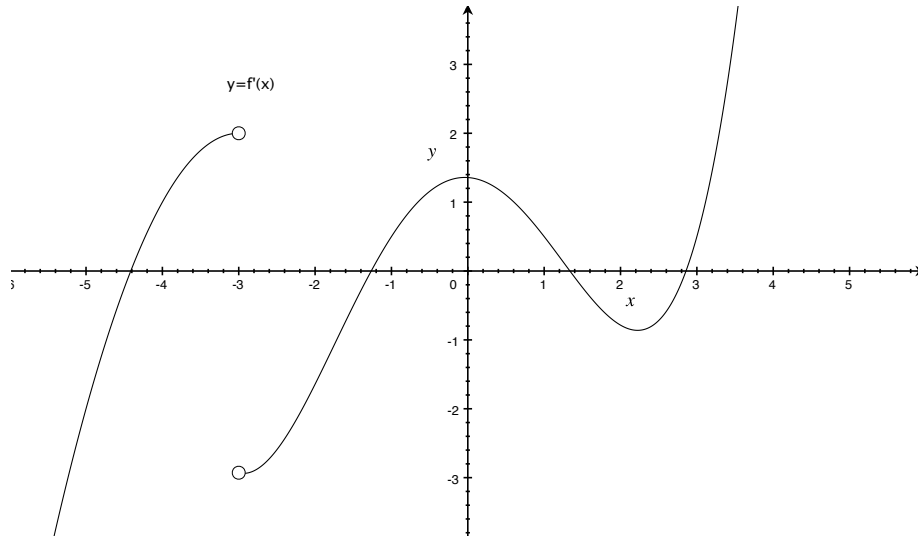
c. (4 pts) Draw and label the graph of the  $f''(x)$  on the graph.

d. (6 pts) At what  $x$  values does  $f(x)$  have local maximum or minimum? Identify which are maximums and which are minimums.

(Hint: Clearly  $f(x)$  is not differentiable at  $x = -3$ . Since  $f(x)$  is continuous that means at  $x = -3$ ,  $f(x)$  turns a sharp corner or comes to a point. We can't define the derivative at such a point (think about the graph of  $|x|$ ). Is  $x = -3$  a local max, min, or neither? Consider it with the others.)

e. (4 pts) At what  $x$ -values does  $f(x)$  have inflection points?

f. (6 pts) Sketch and label **2** possible graphs of  $f(x)$  on the graph below. Remember,  $f(x)$  must be continuous.



5. (8 points) The cubic  $f(x) = \frac{x^3}{3} - ax^2 + bx + c$  has an inflection point at  $x = 1$ . At  $x = 3$  the tangent line to the graph is  $y = 3x - 1$ . Determine  $a$ ,  $b$ , and  $c$ .