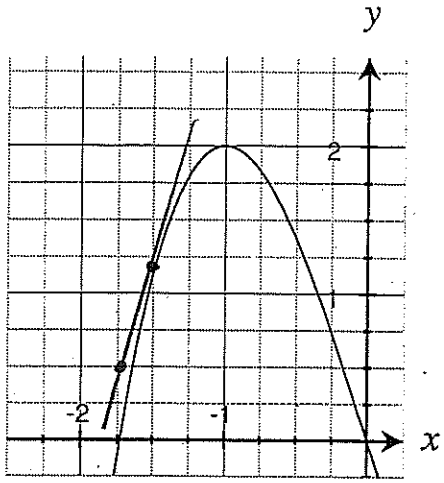
Name: Solutions

Math 105: Fall 2013
Exam 1 (version $\varphi$): October 4

**Correct answers accompanied by incorrect or incomplete work will not receive full credit.**

1. (6 points) A portion of the graph of $g(x) = x^3 - 3x$ is shown




   (a) Draw the line tangent to $g(x)$ at $x = -1.5$.

   (b) Compute the slope of your tangent line.

$(-1.75, 0.5)$, $(-1.5, 1.2)$

$$\frac{0.5 - 1.2}{-1.75 + 1.5} = \frac{-.7}{-.25} = 2.8$$

2. (8 points) Let $g(x) = x^3 - 3x$. Use the limit definition of derivative to find $g'(-1.5)$.
   (Hint: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.)

$$g'(-1.5) = \lim_{h\to 0} \frac{g(-1.5+h) - g(-1.5)}{h}$$

$$= \lim_{h\to 0} \frac{(-1.5+h)^3 - 3(-1.5+h) - [(-1.5)^3 - 3(-1.5)]}{h}$$

$$= \lim_{h\to 0} \frac{(-1.5)^3 + 3(-1.5)^2 h + 3(-1.5)h^2 + h^3 + 4.5 - 3h - (-1.5)^3 - 4.5}{h}$$

$$= \lim_{h\to 0} \frac{6.75h - 4.5h^2 + h^3 - 3h}{h} = \lim_{h\to 0} (3.75 - 4.5h + h^2)$$

$$= 3.75$$

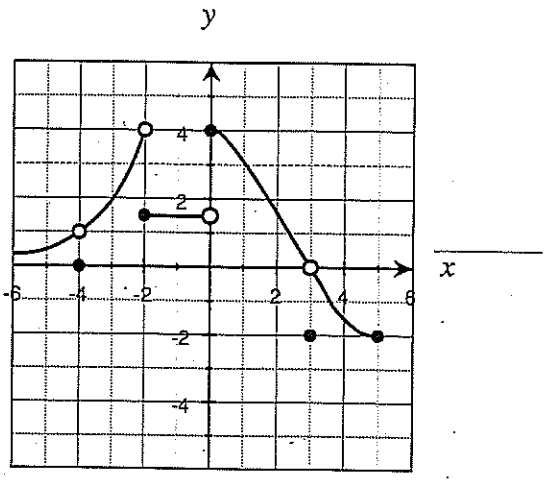
3. (6 points) Let $g(x) = x^3 - 3x$. Use the power rule to find $g'(-1.5)$.

$$g'(x) = 3x^2 - 3 \implies g'(-1.5) = 3.75$$

4. (4 points) Let $H(t)$ be the height (in meters) of a hot air balloon $t$ minutes after take off. What does the statement $H'(60) = -2.4$ mean in this context? Include units in your answer.

60 minutes after take off the balloon is descending at a rate of 2.4 meters/min.

5. (4 points each) The graph of $f(x)$ is given. Evaluate the following (assume the tickmarks occur at 1, 2, etc).

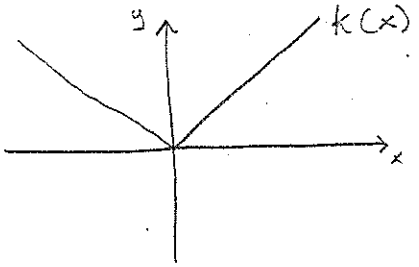



(a) $f(-4) = 0$

(b) $\lim_{x \to -4} f(x) = 1$

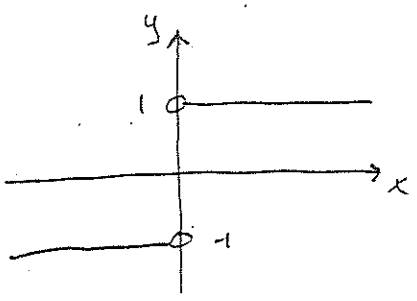
6. (10 points) Let $k(x) = |x|$.

(a) Find $\lim_{x \to 0} k(x)$. Justify your answer.




$\lim_{x \to 0} k(x) = 0$, we can see this by looking at the graph at left.

(b) Find $\lim_{x \to 0} k'(x)$. Justify your answer.




I drew the graph of $k'(x)$ at left.

$\lim_{x \to 0^+} k'(x) = 1$

$\lim_{x \to 0^-} k'(x) = -1$

} since these are not equal, $\lim_{x \to 0} k'(x)$ DNE

7. (14 points) Let $h(x) = 4x^7 + \frac{12}{x^7} - \sqrt[5]{x} + \cos 12$. $= 4x^7 + 12x^{-7} - x^{1/5} + \cos 12$

(a) Find the derivative of $h$.

$h'(x) = 28x^6 - 84x^{-8} - \frac{1}{5}x^{-4/5}$

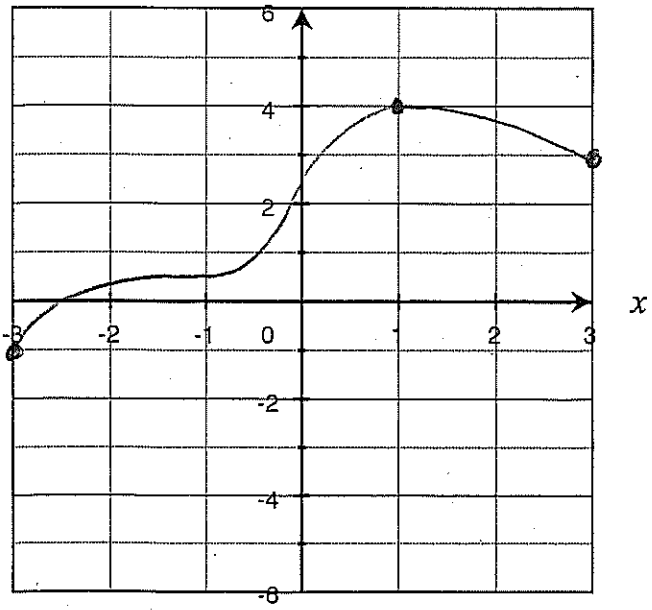
(b) Find the antiderivative of $h$.

$H(x) = \frac{4x^8}{8} + \frac{12x^{-6}}{-6} - \frac{x^{6/5}}{6/5} + (\cos 12)x + C$

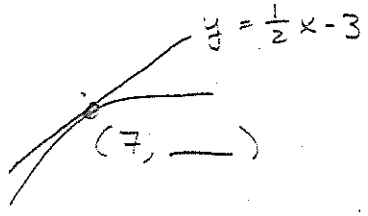
$H(x) = \frac{1}{2}x^8 - 2x^{-6} - \frac{5}{6}x^{6/5} + (\cos 12)x + C$

8. (14 points) Assume that $f$ is a continuous function defined on the closed interval $[-3, 3]$ such that $f(-3) = -1$ and $f(3) = 3$. Furthermore, assume that $f'$ and $f''$ are continuous on $(-3, 3)$ and that the information in the table below is known about these functions. On the grid below sketch $f$.

| $x$ | $-3 \le x < -1$ | $-1$ | $-1 < x < 0$ | $0$ | $0 < x < 1$ | $1$ | $1 < x \le 3$ |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | + | 0 | + | + | + | 0 | − |
| $f''(x)$ | − | 0 | + | 0 | − | − | − |
| $f$ | inc, ccv | terrace | inc, ccv | infl | inc, ccv | max | dec, ccv |



9. (8 points) Assume $y = \frac{1}{2}x - 3$ is tangent to $f(x)$ at $x = 7$.




(a) Find $f(7)$. Justify your answer.

find the y-coordinate when $x=7$

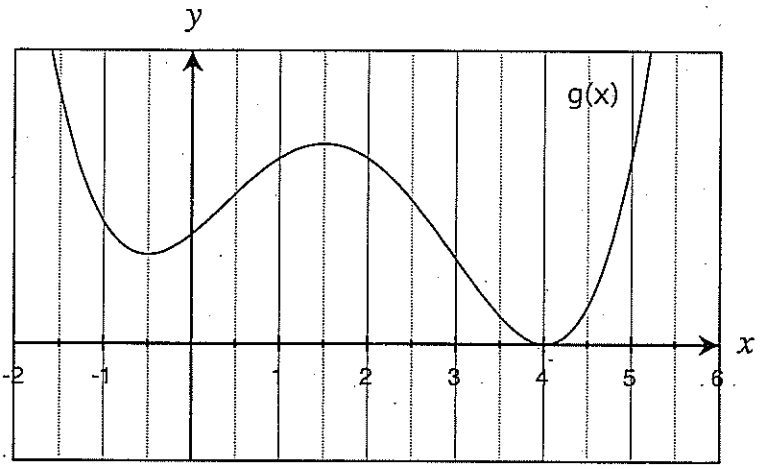
$y = \frac{1}{2}(7) - 3 = \frac{7}{2} - \frac{6}{2} = \boxed{\frac{1}{2}}$

(b) Find $f'(7)$. Justify your answer.

slope of tangent line equals derivative.

so $\boxed{\frac{1}{2}}$ which is the slope of $y = \boxed{\frac{1}{2}}x - 3 = \boxed{m}x + b$

10. (18 points) The graph below is a graph of $g(x)$. Let $G(x)$ be an antiderivative of $g(x)$.




(a) Is it possible that $g''(1.5) = 2$? Justify your answer.

No b/c $g$ is concave down at $x = 1.5$ and the 2nd derivative of $g$ is negative when $g$ is concave down

(b) Is $G(0) > G(1)$? Justify your answer.

$G' = g$, since $g(x)$ is positive for all $0 \le x \le 1$, the slope of $G$ is positive, hence $G$ is increasing on the interval $[0,1]$. So no, instead $G(0) < G(1)$

(c) On what interval(s) is $G(x)$ concave down? Justify your answer.

$G(x)$ is cc $\downarrow$ when $G' = g$ is decreasing.

So $(-\infty, -0.5) \cup (1.5, 4)$

11. (4 points) Who do you think will win the World Series?

(a) Atlanta Braves (b) Boston Red Sox (c) Detroit Tigers (d) Los Angeles Dodgers
(e) Oakland Athletics (f) Pittsburgh Pirates (g) St. Louis Cardinals (h) Tampa Bay Rays