Math 205 Quiz 4
Name:

1. Let A be an $m \times n$ matrix, B be an $n \times p$ matrix, and C be an $n \times m$ matrix where $n \neq m \neq p$.

   (a) Circle the matrix multiplications that can be computed.

   $$\boxed{\text{AB}} \qquad BC \qquad CB \qquad \boxed{\text{CA}} \qquad \boxed{\text{AC}} \qquad A^TB \qquad A^TC \qquad \boxed{B^TC}$$

2. Consider the matrix $A = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & 0 \\ 0 & -4 & 1 \end{pmatrix}$.

   (a) Find the inverse of A.

   $$\left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & -4 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{R3=R2+R3} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right) \xrightarrow{1/4*R2} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right)$$

   $$\xrightarrow{R1=2*R2+R1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 1/4 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right) \text{ then } A^{-1} = \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 1/4 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

   (b) Row reducing A to the Identity can be done in three steps. Therefore there are three elementary matrices, E_1, E_2, and E_3 such that $E_3E_2E_1A = I$. Determine these matrices. Order matters. E_1 is the first row operation to be performed, etc.

   $$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$
   $$E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
   $$E_3 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

   These are not unique. If you did a different order or different operations, then you have different E_1, E_2, E_3.

3. Consider the matrix $A = \begin{pmatrix} 4 & h \\ h & 1 \end{pmatrix}$.

   (a) For what value(s) of h will A^{-1} fail to exist?

   $\det(A) = 4 - h^2$ and the determinant cannot be zero. So, $h \neq \pm 2$.

   (b) Determine the inverse of the matrix A when it exists.

   $$\frac{1}{4-h^2}\begin{pmatrix} 1 & -h \\ -h & 4 \end{pmatrix}$$