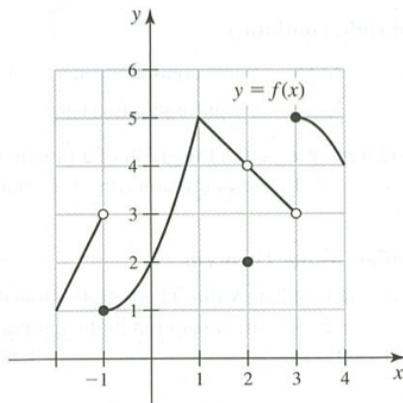Name:_____

Exam 1 - Math 105C

Show all your work to receive full credit for a problem. There are a total of 72 points on this test.

1. (7 points) The graph of a function f is given below. Use the graph to determine the values of x in the interval $(-2, 4)$ at which f is not continuous. Justify your answers using the language of limits (that is, I don't want you to just make references to the picture).


2. (6 points) The following limit is a derivative, but of what function f and at what point?

$$\lim_{h \to 0} \frac{4(2+h)^3 - 4(2)^3}{h}$$

3. (4 points each, 12 points total) The population of the United States (in millions) by decade is given in the table below, where t denotes the number of years after 1900. These data are plotted and fitted with a smooth curve $y = p(t)$ in the figure.

Year	1940	1950	1960	1970	1980	1990	2000	2010
t	40	50	60	70	80	90	100	110
$p(t)$	132.16	152.32	179.32	203.30	226.54	248.71	281.42	308.94
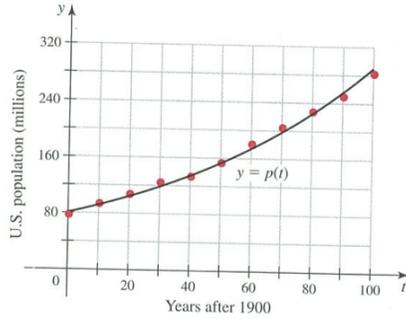
(a) Compute the average rate of population growth from 1950 to 1960.

(b) Explain why the average rate of growth from 1950 to 1960 is a good approximation to the (instantaneous) rate of growth in 1955.

(c) Estimate the instantaneous rate of growth in 1985.

4. (3 pts each, 18 pts total) Are the following statements true or false? If a statement is true, give a brief explanation or an example illustrating it. If a statement is false, give a counterexample.

(a) If the tangent line to the graph of $y = f(x)$ is horizontal at $x = c$, then $f'(c) = 0$.

(b) If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.

(c) If $f'(c) = 0$ then c has to be either a local maximum or a local minimum of $f(x)$.

(d) If f is continuous at c, then f is differentiable at c.

(e) If the acceleration of an object is negative, then its velocity is decreasing.

(f) The equation of the line tangent to the graph of $y = x^3$ at $(1, 1)$ is $y - 1 = 3x^2(x - 1)$.

5. (3 pts each, 15 pts total) Below is the graph of f', the derivative of a function f. At which of the points $0, x_1, x_2, x_3, x_4, x_5$, is the function f (justify all of your answers):
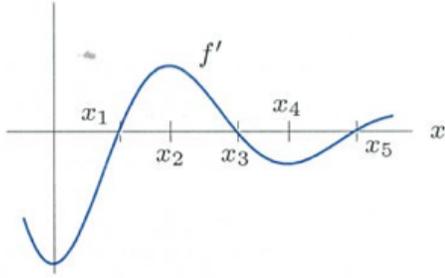
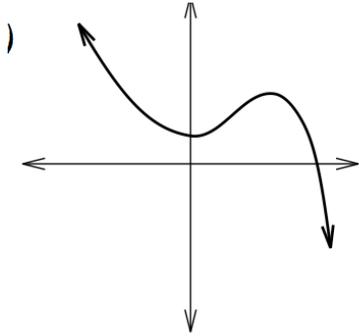(a) At a local maximum value?

(b) At a local minimum value?

(c) At an inflection point?

(d) Increasing the fastest?

(e) Decreasing the most?

6. (7 pts each, 14 pts total) Below is the graph of a function g.

(a) Draw a graph of $g'(x)$, the derivative of $g(x)$.

(b) Draw a graph of $G(x)$, an antiderivative of $g(x)$.