

Mid-term Exam #1
MATH 106-D, Fall 2015

Name: _____

Instructions:

- Please answer as many of the following questions as possible.
- No cell phones or collaboration allowed. If you leave the classroom during the exam you must leave your cell phone with the instructor.
- Approved calculators are allowed.
- Additional scrap paper is available upon request.
- *Multiple choice questions:* Circle the letter corresponding to your answer. No partial credit will be awarded.
- *Short answer questions:* Show all of your work on the page of the problem. Clearly indicate your answer and the reasoning that you used to arrive at the answer. You do not have to simplify algebraic expressions.

This exam has 4 multiple choice problems and 5 short answer problems. There are a total of 100 points.

Good luck!

Problem	Possible Points	Points Earned
MC	20	
5	24	
6	18	
7	12	
8	16	
9	10	
TOTAL	100	

1. (5 points) Let $I = \int_a^b f(x) dx$ and suppose that $f(x)$ is increasing and concave down on the interval $[a, b]$. Which of the following statements is true?

A. $L_n \leq I \leq T_n$

B. $L_n \leq I \leq R_n$

C. $I \leq L_n \leq R_n$

D. $R_n \leq L_n \leq I$

E. $T_n \leq I \leq L_n$

2. (5 points) Which of the following integrals describes the volume of the solid obtained by revolving the region between $y = 1$ and $y = 4 - x^2$ around the x -axis?

A. $\int_{-\sqrt{3}}^{\sqrt{3}} \pi (3 - x^2)^2 dx$

B. $\int_{-\sqrt{3}}^{\sqrt{3}} \pi ((4 - x^2)^2 - 1) dx$

C. $\int_{-2}^2 \pi ((4 - x^2)^2 - 1)^2 dx$

D. $\int_{-2}^2 \pi (4 - x^2)^2 dx$

E. $\int_{-\sqrt{3}}^{\sqrt{3}} 2\pi (4 - x^2 - 1) dx$

3. (5 points) The length of the curve $y = f(x)$ from $x = 2$ to $x = 5$ is given by the integral $\int_2^5 \sqrt{(\sqrt{1+x})^2 + 1} dx$. Choose the appropriate $f(x)$.

A. $f(x) = \sqrt{1+x}$

B. $f(x) = (1+x)^{3/2}$

C. $f(x) = 1+x$

D. $f(x) = \frac{2}{3}(1+x)^{3/2}$

E. $f(x) = (\sqrt{1+x})^2 + 1$

4. (5 points) Which of the following integrals describes the area of the region bounded by the curves $y = (x-2)^3 + 2$ and $y = x$?

A. $\int_1^2 x - (x-2)^3 + 2 dx + \int_2^3 (x-2)^3 + 2 + x dx$

B. $\int_1^3 x - ((x-2)^3 + 2) dx$

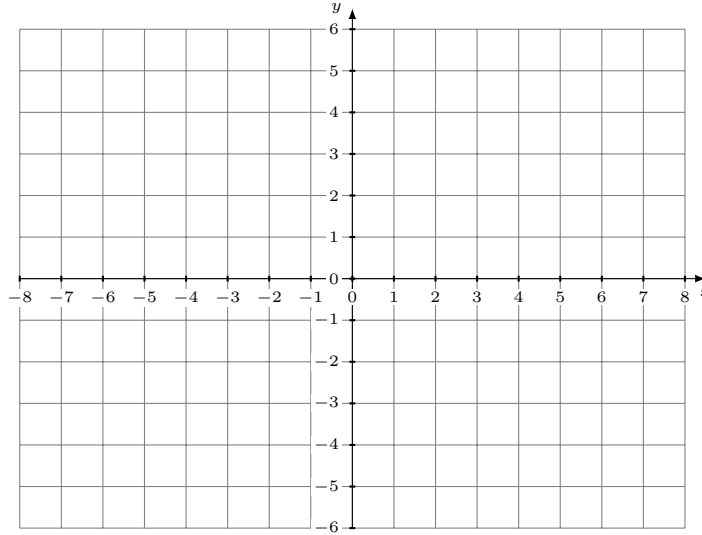
C. $\int_1^3 (x-2)^3 + 2 - x dx$

D. $\int_1^2 (x-2)^3 + 2 dx - \int_2^3 x dx$

E. $\int_1^2 (x-2)^3 + 2 - x dx + \int_2^3 x - ((x-2)^3 + 2) dx$

5. (24 points)

- (a) (4 points) Draw the region enclosed by the line $y = x + 2$ and the parabola $y = x^2$ and label the points of intersection.



- (b) (10 points) Find the volume of the solid obtained by rotating the region in part (a) around the line $y = -4$.

(c) (5 points) Set up (**but do not evaluate**) an integral with respect to x that computes the area of the enclosed region.

(d) (5 points) Set up (**but do not evaluate**) an integral with respect to y that computes the area of the enclosed region.

6. (18 points) Compute the following integrals.

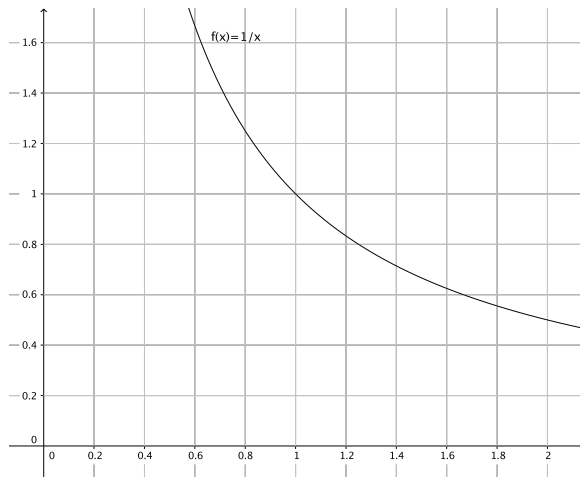
(a) (6 points) $\int (e^x + 1)\sqrt{2e^x + 2x} dx$

(b) (6 points) $\int \frac{1}{x^2 - 2x + 1} dx$

(c) (6 points) $\int_0^{\sqrt{\pi/2}} x \cos(x^2) dx$

7. (12 points) The identity $\int_1^2 \frac{1}{x} dx = \ln(2)$ gives us a way to approximate $\ln(2)$.

(a) (4 points) Draw M_4 on the plot below and compute M_4 , the midpoint approximation of $\ln(2)$ with 4 subdivisions.



$$M_4 =$$

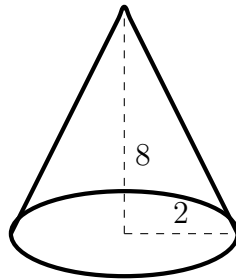
(b) (8 points) Let $I = \int_a^b f(x) dx$. Then the error bound for Simpson's rule is the following: Let K_4 be a constant such that $|f^{(4)}(x)| \leq K_4$ for all x in $[a, b]$. Then

$$|I - S_{2n}| \leq \frac{K_4(b-a)^5}{180(2n)^4}.$$

Here, $f^{(4)}(x)$ is the fourth derivative of f .

When $f(x) = \frac{1}{x}$, $f^{(4)}(x) = \frac{24}{x^5}$. Using this fact and the rule stated above, how many subdivisions are required to obtain a Simpson's rule approximation of $\ln(2)$ with error at most 0.000005? *You can give your answer in terms of n or $2n$.*

8. (16 points) A tank has the shape of a right circular cone with height 8 feet and base radius 2 feet.



- (a) (4 points) Compute the area of a (horizontal) cross section of the tank at height y .
- (b) (4 points) Suppose that the tank is filled with olive oil to a height of 4 feet. How many cubic feet of olive oil are in the tank?
- (c) (8 points) Suppose that the tank is filled with olive oil to a height of 4 feet. How much work does it take to empty the tank by pumping all of the olive oil to the top of the tank? Olive oil weighs 57 lb/ft^3 .

9. (10 points) Solve the initial value problem

$$\frac{dx}{dt} = \frac{1-t^2}{x^2}, \quad x(1) = 1.$$