

Math 106A/B  
Calculus 2  
Fall 2015  
Exam 1  
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Please show your work.

1. The graph of a function,  $f(x)$ , is increasing and concave down on the interval  $[a,b]$ . Put the following quantities in **increasing** order:

$$L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) dx.$$

2. Evaluate by finding an antiderivative. [Your final answer should not contain an integral.]

a)  $\int (x^2 - 9)^3 dx$

b)  $\int \frac{\cos(x)}{1 - \sin(x)} dx$

3. Evaluate:  $\int_0^1 \frac{1}{x^2 - 4x + 4} dx$

4. A cylindrical tank of radius 4 ft. and height 10 ft is filled halfway with water. Find the work required to pump all the water over the upper rim.

5. Find the arclength of a function on the interval  $[2,3]$  where the derivative of the function is given by

$$f'(x) = \sqrt{x^2 - 1}.$$

6. Consider the functions  $f(x) = \frac{x^2}{4}$  and  $g(x) = x$ .

a) Draw the graphs of  $g(x)$  and  $f(x)$ .

b) Using integration, find the area between the two functions.

c) Find the volume of the solid formed by revolving the area between the functions around the x-axis.

7. Find the solution of the initial value problem:

$$\frac{dy}{dx} = x(1+y^2) \text{ with } y(0) = 1.$$

8. Let  $I = \int_0^1 \frac{4}{1+x^2} dx$ . If we were to calculate the integral exactly, we would find that  $I = \pi$ . Therefore, we can use this result and numerical integration techniques to approximate the value of  $\pi$ .

a) Compute  $L_4$ , the left sum approximation with 4 subdivisions.

b) How many subdivisions are required to obtain a left sum approximation with error of at most  $1/10,000$  Recall that the error bound estimate for left sums may be determined using

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}.$$