

**MATH 205A,B - LINEAR ALGEBRA
FALL 2015**

QUIZ 3

NAME: _____ **Section:**(Circle one) A(8 : 00) B(9 : 30)

Show **ALL** your work **CAREFULLY**.

Let

$$A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}.$$

(a) Solve the homogeneous equation $A\vec{x} = \vec{0}$.

The corresponding augmented matrix

$$\begin{bmatrix} 1 & -3 & 0 \\ 3 & 5 & 0 \\ -1 & 7 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 3 & 5 & 0 \\ 0 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 0 & 14 & 0 \\ 0 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The solution is $x_1 = 0 = x_2$.

(b) Based on your answer to (a), determine whether the columns of A are linearly independent? Justify your answer.

Since $A\vec{x} = \vec{0}$ has only the trivial solution, the zero vector can only be the trivial linear combination of the columns of A . Hence the columns of A are linearly independent.

(c) Do the columns of A span \mathbb{R}^3 ? Explain.

No. Since there are only two vectors (columns), the span of two vectors can be at most \mathbb{R}^2 so the columns of A do not span \mathbb{R}^3 .

Alternatively, the equation $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has no solution (CHECK!) so the vector $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ does not lie in the span of the columns of A .