Math 205 A B Quiz 03 page 1 09/28/2012 **Name** suggested sol'n

Let $A = \begin{bmatrix} 0 & 2 & 2 & 4 & 2 & 4 \\ 1 & 6 & 3 & 9 & 2 & 3 \\ -2 & 3 & 4 & 12 & 3 & 8 \\ 3 & 6 & 0 & 3 & -1 & -5 \\ 1 & 2 & -1 & 1 & -1 & -3 \end{bmatrix}$; then rref($A$)= $\begin{bmatrix} 1 & 0 & 0 & -3 & 0 & -1 \\ 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Let $S = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_6\}$ be the set of column vectors of A.

1. Write all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 3x_4 + x_6 \\ -2x_4 + 0x_6 \\ 0 \\ x_4 \\ 0x_4 - 2x_6 \\ x_6 \end{bmatrix} = x_4 \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$\underbrace{\text{where } x_4 \text{ and } x_6 \text{ are free.}}_{\text{always write this at the end!}}$

2. It's clear that S is LD. Which vectors in S *can* be expressed as linear combinations (LC's) of the other vectors? Don't write the vectors out; use their names ($\mathbf{a}_1$, $\mathbf{a}_2$, etc.) in a simple list instead.

According to (1), we'll be able to find ways to write $\vec{0}$ as $x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_6\vec{a}_6 = \vec{0}$ where x_1, x_2, x_4, x_5, x_6 don't need to be 0; so each of the corresponding vectors $\vec{a}_1$ $\vec{a}_2$ $\vec{a}_4$ $\vec{a}_5$ and $\vec{a}_6$ can be written as LC's of the others (but not $\vec{a}_3$ ... see 4 below!)

3. If $\mathbf{a}_2$ can be written as a LC of the other vectors, find a way to do it using as few of the other vectors as possible. Otherwise, explain correctly why $\mathbf{a}_2$ is not a LC of the other vectors.

the idea is to find a sol'n of $A\vec{x} = \vec{0}$ in which the weight in front of $\vec{a}_2$ is <u>NOT</u> 0.

try $x_4 = 1$ and $x_6 = 0$. Then the other weights are $x_1 = 3\cdot 1 + 0$

$x_2 = -2\cdot 1 + 0$

$(x_3 = 0 \text{ always})$

$x_5 = -2\cdot 0$

so $3\vec{a}_1 + -2\vec{a}_2 + 0\vec{a}_3 + 1\vec{a}_4 + 0\vec{a}_5 + 0\vec{a}_6 = \vec{0}$

or, $3\vec{a}_1 - 2\vec{a}_2 + \vec{a}_4 = \vec{0}$. Solve for $\vec{a}_2$ to find

$\vec{a}_2 = \frac{1}{2}(3\vec{a}_1 + \vec{a}_4)$; finally then $\boxed{\vec{a}_2 = \left(\frac{3}{2}\right)\vec{a}_1 + \left(\frac{1}{2}\right)\vec{a}_4}$

shows how to write $\vec{a}_2$ as a LC of $\vec{a}_1$ & $\vec{a}_4$.

4. If $\mathbf{a}_3$ can be written as a LC of the other vectors, find a way to do it using as few of the other vectors as possible. Otherwise, explain correctly why $\mathbf{a}_3$ is not a LC of the other vectors.

The <u>vector</u> $\vec{a}_3$ cannot be written as a LC of the other vectors.

For suppose it CAN BE. then $\vec{a}_3 = $ (some LC of the other vectors)

so $\vec{a}_3 = x_1\vec{a}_1 + x_2\vec{a}_2 + x_4\vec{a}_4 + x_5\vec{a}_5 + x_6\vec{a}_6$ for some weights $x_1, x_2, \ldots, x_6$

subtract $\vec{a}_3$ from both sides: $\vec{0} = x_1\vec{a}_1 + x_2\vec{a}_2 + (-1)\vec{a}_3 + x_4\vec{a}_4 + x_5 a_5 + x_6\vec{a}_6$

<u>BUT</u> our solution of $A\vec{x} = \vec{0}$ in (1) tells us that in ANY sol'n of $A\vec{x} = \vec{0}$, the weight x_3 of $\vec{a}_3$ is 0, yet here's a sol'n with $x_3 = -1$, a contradiction, since x_3 can't be both 0 and -1. So $\vec{a}_3$ is <u>NOT</u> a LC of the others.