Math 106 Name _____
Exam 1
9/28/12

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. In other words, correct answers accompanied by incorrect or incomplete work will not receive full credit.

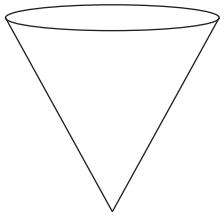
1. Evaluate the following integrals.

   (a) $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

   (b) $\displaystyle\int_0^1 \frac{3}{(2-x)^2}\,dx$

   (c) $\displaystyle\int \sqrt{1+\sqrt{x}}\,dx$

2. Consider a 15 ft tall conical tank with base radius of 5 ft, oriented as shown below. Suppose the tank is filled to 4 ft below the rim with benzene weighing 56 lb/ft^3. *Set up, but do not evaluate* an integral representing the work done pumping the benzene to a level 3 ft above the top of the tank.



3. (a) Find a function whose arc length is given by $\int_1^2 \sqrt{1+\frac{1}{x^4}}\,dx$. Justify your answer.

(b) If we don't view the integral in (a) as an arc length, what other interpretation might we assign to this integral?

4. Measurements are made of the rate at which water is draining from a container at various times, and recorded in the following table.

t (min)	0	0.5	1	1.5	2
$r(t)$ (liters/min)	17	16	14	10	2

(a) In a sentence or two, explain why someone would want to calculate $\int_0^2 r(t)\,dt$. What is the meaning of this integral?

(b) Which of the *left*, *right*, *midpoint*, and *trapezoid* sums should you choose to produce the best approximation you can to the value of $\int_0^2 r(t)\,dt$. Calculate the sum you selected, be sure to show enough work so that your method is clear. Provide a sentence to explain why you think this is the best approximation given the limitations of the table.

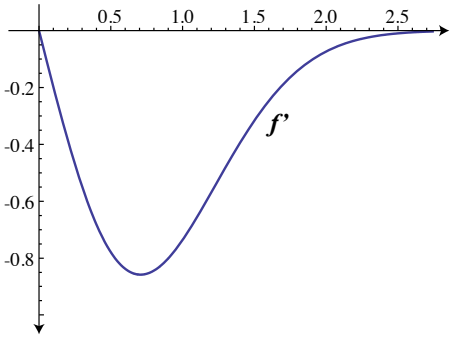
5. Consider the region bounded by the curves $y = x^2 - 4$ and $y = -x^2 - 2x$.

(a) Sketch the region described above. Find and label the intersection points.

(b) Find the area of the region described above.

(c) *Set up, but do not evaluate,* an integral which represents the volume of the solid formed by revolving the region around the line $y = 2$.

6. Consider a function f where the graph of its *derivative* f' is shown below. Let $I = \int_1^2 \ln(f(x))\,dx$.



(a) Suppose the only thing you know about f is that $f(2) = 0.018$, but you wish to use R_n to estimate I. Determine if R_n is an overestimate or underestimate for the exact value of I. Justify your answer. (Hint: carefully consider the integrand as well as characteristics about f'.)

(b) How many subdivisions are required to obtain a right sum approximation with error of at most 1/10,000? Recall that the error bound estimates for right sums may be determined using:

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}.$$