MATH106B,C CALCULUS II - PROF. P. WONG

EXAM I - SEPTEMBER 27, 2013

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	22	
5.	18	
Total	100	

1.(10 pts.)(a) Evaluate the indefinite integral (be sure to show all your work)

$$\int (\sin x) e^{\cos x}\, dx.$$

(10 pts.) (b) Find the **exact value** of the definite integral (be sure to show all your work)

$$\int_0^2 \frac{1+e^x}{x+e^x}\, dx.$$

2. Consider the region A in the first quadrant bounded by the curve $y = x^3$ and the curve $y = x(2 - x)$.

(15 pts.) Find the **exact area** of the region A.

(5 pts.) The same two curves $y = x^3$ and $y = x(2 - x)$ also bound a region B in the third quadrant. Write a definite integral (do not evaluate) representing the area of region B. [By area, we mean the usual *physical* area not *signed* area.]

3. (12 pts.) Consider a function h on the interval $[0, 2]$.

x	0	0.5	1	1.5	2
$h(x)$	-1	1	3	2	-1

Find L_4, T_4 using the left-hand sum and the trapezoid rule respectively for estimating the definite integral $\int_0^2 h(x)\ dx$.

(8 pts.)(b) Recall that the error committed by using the right hand sum approximation R_n is less than or equal to $\frac{K_1 \cdot (b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant K_1 over the interval $[a, b]$. Use this result to give an upper bound for the error committed by R_{10} for

$$I = \int_1^3 (\sin x)(\ln x)\ dx.$$

4. Let R be the region bounded by the curve $y = \frac{2}{x}$, the line $y = 1$, the line $y = 2$, and the line $x = 1$.

(12 pts.) (a) Set up (do not evaluate) a definite integral representing the volume of the solid obtained from rotating the region R around the line $x = 0$, i.e., the y-axis. [Hint: sketch a picture of the region R first.]

(10 pts.) (b) Find the **exact volume** of the solid described in part (a).

5. Consider the initial value problem

$$\frac{dy}{dx} = (1 + y^2)e^x$$

with $y(0) = 0$.

(10 pts.)(a) Use the technique of separation of variables to solve the Initial Value Problem.

(8 pts.)(b) Set up (do not evaluate) a definite integral for the arc length of the portion of the graph of $f(x) = x \ln x$ between $x = 1$ and $x = e$.