Name: Solutions

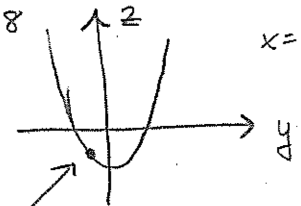
Math 206: Fall 2013
Exam 1: September 27

calculator allowed

Good Luck!

1. (12 points) Consider the function $z = f(x, y) = 3y^2 - 4x^3$. Suppose you are standing on the surface at the point where $x = 3$ and $y = -1$. If you start to move on the surface parallel to the y-axis in the direction of increasing y, does your height increase or decrease? Explain your answer

x fixed: $f(3, y) = 3y^2 - 108$



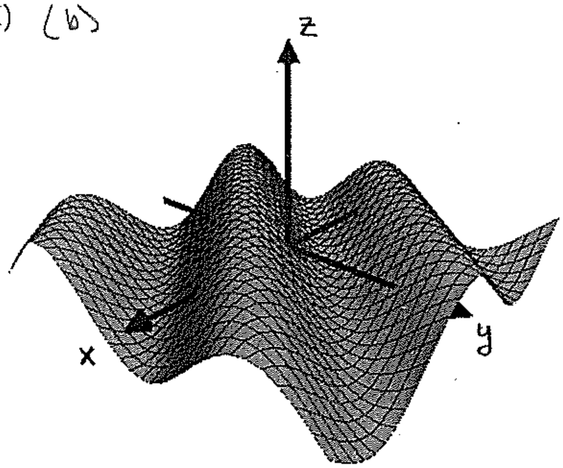
$x=3$ cross-section

from here as y increases $\boxed{z \text{ decreases}}$

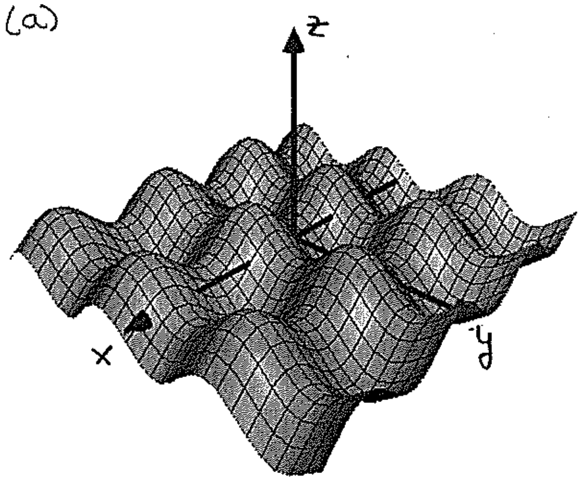
2. (16 points) Match the function with its graph. Give brief reasons for your choices. (Your reason should NOT be "because that's what my calculator graphed".)

   (a) $z = \sin x - \sin y$ (II)
   (b) $z = \sin(x - y)$ (I)

(I) (b)



(II) (a)

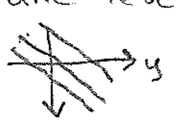


fix $y = k \Rightarrow$ (a) $z = \sin x - \sin k$. So the basic $z = \sin x$ graph is shifted up and down as y varies.
(b) $z = \sin(x-k)$. So the basic $z = \sin x$ graph is shifted left and right as y varies

Thus (a) $\to$ (II) and (b) $\to$ (I)

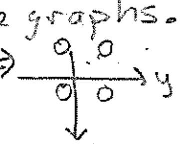
Alternatively fix $z = k$; take level sets of the graphs.

(I) $\Rightarrow$



seem to be // lines for all k.

(II) $\Rightarrow$

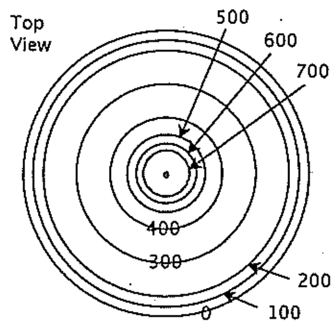


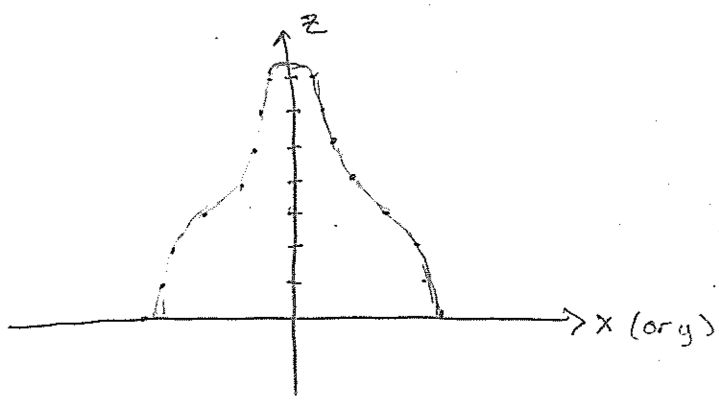
seem to be circles around peaks

(b) $z = k \Rightarrow k = \sin(x-y) \Rightarrow$ for all k, $x - y =$ (a number) $\Rightarrow$ $y = x +$ (number) which are // lines like we drew above.

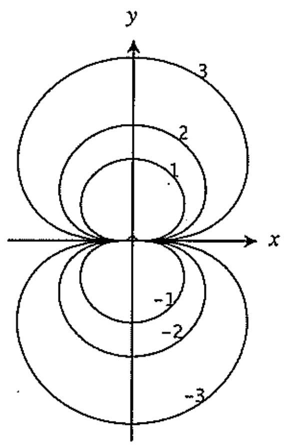
Thus (I) $\to$ (b) and by default (II) $\to$ (a)

3. (12 points) The diagram below shows the contour map for a circular island. (Units are in feet.) Sketch the vertical cross-section of the island that passes through the island's center.



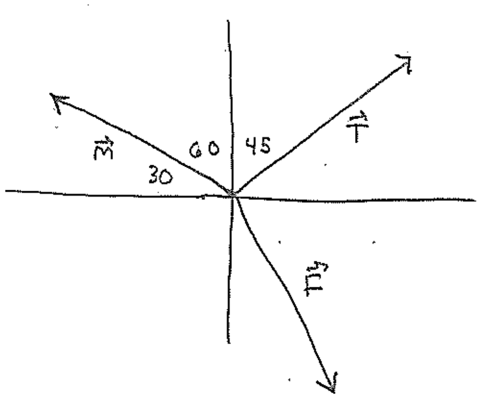


4. (12 points) A function $f(x, y)$ is defined for $(x, y) \neq (0, 0)$. Is $\lim_{(x,y)\to(0,0)} f(x, y)$ likely to exist if f has the contour diagram below? Explain your answer.



No. As we approach (0,0) along different paths (namely along different contour curves) then f(x,y) approaches (stays at) the value on the contour curve. Since $\lim_{(x,y)\to(0,0)} f(x,y)$ is different for different paths, the limit doesn't exist.

5. (12 points) Three people are trying to hold a ferocious lion still for the veterinarian. The lion, in the center, is wearing a collar with three ropes attached to it and each person has hold of a rope. Malaika is pulling in the direction 60° west of north with a force of 380 pounds and Thurston is pulling in the direction 45° east of north with a force of 400 pounds. What is the force needed on the third rope to counterbalance Malaika and Thurston? (Your answer will be a vector.)



(guess at $\vec{F}$'s location)

$\|\vec{M}\| = 380$ $\qquad$ $\vec{F} + \vec{M} + \vec{T} = \vec{0}$

$\|\vec{T}\| = 400$

$\vec{T} = 200\sqrt{2}\,\hat{\imath} + 200\sqrt{2}\,\hat{\jmath} = 282.8\,\hat{\imath} + 282.8\,\hat{\jmath}$

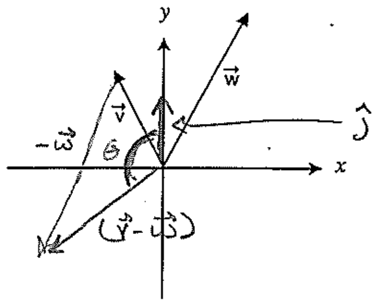
$\vec{M} = -190\sqrt{3}\,\hat{\imath} + 190\,\hat{\jmath} = -329.1\,\hat{\imath} + 190\,\hat{\jmath}$

$\vec{F} = (190\sqrt{3} - 200\sqrt{2})\,\hat{\imath} - (200\sqrt{2} + 190)\,\hat{\jmath}$

$= (329.1 - 282.8)\,\hat{\imath} - (282.8 + 190)\,\hat{\jmath}$

$= 46.3\,\hat{\imath} - 472.8\,\hat{\jmath}$

6. (12 points) The vectors $\vec{v}$ and $\vec{w}$ are shown below. Determine whether the following statement is true or false: $(\vec{v} - \vec{w}) \cdot \hat{j} > 0$. Justify your answer.



$(\vec{v} - \vec{w}) \cdot \hat{j} = \|\vec{v} - \vec{w}\| \, \|\hat{j}\| \cos\theta$

$\theta > 90 \Rightarrow \cos\theta < 0$

$\Rightarrow (\vec{v} - \vec{w}) \cdot \hat{j} < 0$

so the statement is $\boxed{\text{false}}$

7. (12 points) Find an equation for the plane passing through the point $(1, -5, -2)$ and containing the x-axis.

b/c the plane contains the x-axis, $\frac{\Delta z}{\Delta x} = 0$ = slope in x-dirn

$(1,0,0)$ is on x-axis. using this point and $(1,-5,-2)$

we have $\frac{\Delta z}{\Delta y} = \frac{-2}{-5} = \frac{2}{5}$ = slope in y-dirn

b/c the plane contains the x-axis the z-intercept is 0

so

$$z = 0x + \frac{2}{5}y + 0 \Longrightarrow \boxed{z = \frac{2}{5}y}$$

8. (12 points) For what value(s) of a is the vector $\vec{v} = 3\hat{i} + a\hat{j} + 3\hat{k}$ parallel to the plane $z = 2x - 5y + 2$?

$\downarrow$

$2x - 5y - z + 2 = 0$

the plane has normal vector

$n = 2\hat{i} - 5\hat{j} - \hat{k}$

$\vec{v}$ is // to plane if $\vec{v}$ is $\perp$ to $\vec{n}$

so we want $\vec{v} \cdot \vec{n} = 0$

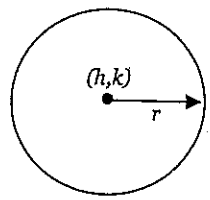
$\vec{v} \cdot \vec{n} = 3 \cdot 2 + a(-5) + 3(-1)$

$= 3 - 5a = 0$

$$\boxed{a = \frac{3}{5}}$$

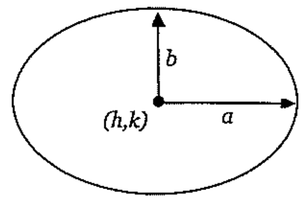
Conic Sections

circle



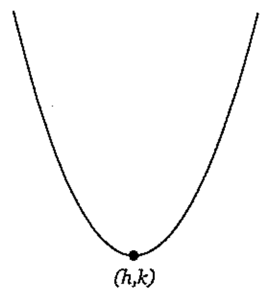
- $(x-h)^2 + (y-k)^2 = r^2$
- center at (h, k)
- radius $= r$

ellipse



- $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
- "center" at (h, k)
- length of x-axis $= 2a$
- length of y-axis $= 2b$

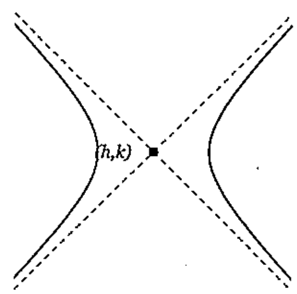
parabola



- $y = a(x-h)^2 + k$
- "vertex" at (h, k)
- $a > 1$ stretches the parabola
- $1 > a > 0$ squishes the parabola

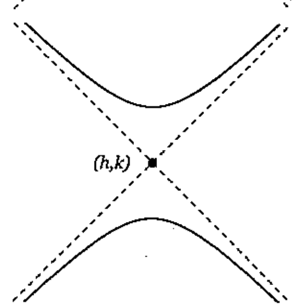
hyperbola

"open sideways"



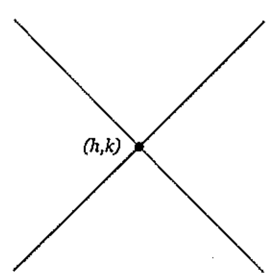
- $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
- "center" at (h, k)
- equations of asymptotes are $y = k \pm \frac{b}{a}(x-h)$

"open up and down"



- $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$
- "center" at (h, k)
- equations of asymptotes are $y = k \pm \frac{a}{b}(x-h)$

"degenerate"



- $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 0$
- "hyperbola" is an $\times$
- "center" at (h, k)
- equations of lines are $y = k \pm \frac{a}{b}(x-h)$