Math 205 Quiz 3
Name:

1. Consider the linear transformation $T(x_1, x_2, x_3) = (x_1 - 2x_2, x_1 + x_2 + x_3)$.
   (a) What is the domain of T? $\mathbb{R}^3$
   (b) What is the codomain of T? $\mathbb{R}^2$
   (c) What is the image of $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

$$\begin{bmatrix} -3 \\ 6 \end{bmatrix}$$

   (d) Determine the matrix A such that $T(\vec{x}) = A\vec{x}$.

$$T(\vec{e_1}) = (1, 1), T(\vec{e_2}) = (-2, 1), T(\vec{e_3}) = (0, 1)$$

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

   (e) Is T one-to-one? Briefly explain.
   No, the matrix A has a free variable, so there may be infinite many $\vec{x}$ sent to a $\vec{b}$ in $\mathbb{R}^2$.
   (f) Is T onto? Briefly explain.
   Yes, the matrix A has a pivot in every row. So the columns of A span $\mathbb{R}^2$.

2. Provide a brief written answer to the following.
   (a) The matrix equation $A\vec{x} = \vec{b}$ is inconsistent if and only if rref($[A \mid \vec{b}]$)
   There is a row [000000|1] or a pivot in the last column..
   (b) What is the definition of the **span** of a set of vectors?
   The span is the set of all linear combinations of the vectors.
   (c) If a set of vectors is **linearly dependent**, then what does that mean? NOT how can you tell, what does it mean?
   That means that at least one of the vectors is a linear combination of the others.
   (d) How can you tell if a set of vectors is **linearly dependent**?
   You can solve the system $A\vec{x} = \vec{0}$ and look for a free variable. This means there exists a non-trivial solution and you can write a linear dependence relation.
   (e) Give one statement that is equivalent to: "Let A be an $m \times n$ matrix. The matrix equation $A\vec{x} = \vec{b}$ has a solution for every $\vec{b}$ in $\mathbb{R}^m$."
      i. Every row of A has a pivot.
      ii. The columns of A span $\mathbb{R}^m$.
      iii. Every $\vec{b}$ in $\mathbb{R}^m$ can be written as a linear combination of columns of A.
      iv. The transformation $T(\vec{x}) = A\vec{x}$ is onto.