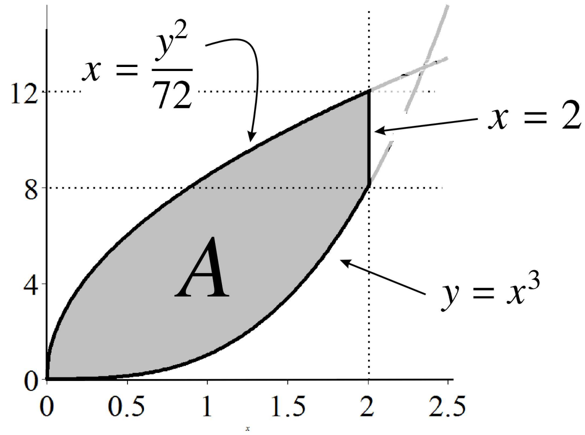
1. Consider the shaded region A in the plane of all points which are between the graphs of $x = \dfrac{y^2}{72}$ and $y = x^3$ for $0 \le x \le 2$. The region A is shown to the right.



1A. Set up the integral which represents the area of A if the corresponding LHS approximations use rectangles each of whose base width is $\triangle x$ and each rectangle goes from a top curve down to a bottom curve. That is, the integral is of the form $\int dx$.

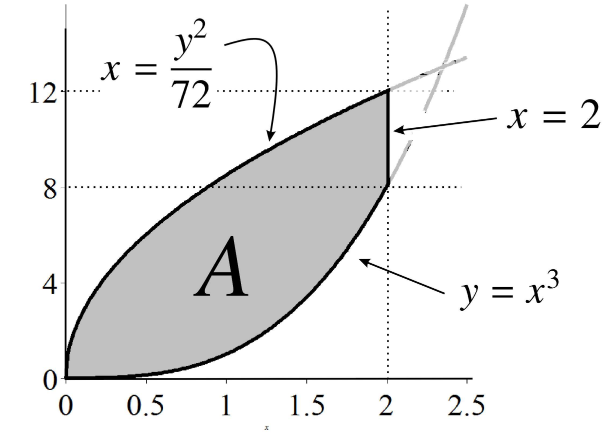
1B. BONUS!!! (Work on this only if you have time!) Use the fundamental theorem of calculus to find the exact value of the integral in 1A. You may find it useful to write $\sqrt{72x}$ as $\sqrt{72}\, x^{1/2}$ when finding the antiderivative you need. Near the end you may also find it useful that $\sqrt{72}\sqrt{2^3}$ is just 24. *Show all your work. Simplify the answer.*

2A. Set up the integral (in terms of dx) that represents the arc-length of the top curve from $(0, 0)$ to $(2, 12)$ in the figure above.

2B. Find the MID(50) approximation to the integral in 2A to as many places as your calculator displays.

2C. What is the straight-line distance between $(0, 0)$ and $(2, 12)$? (Show your work)

3. Again, consider the shaded region A from problem 1, shown here again for convenience.



3A. Set up the integral(s) which represents the volume of the solid of revolution obtained by revolving A around the line $y = 12$. All expressions representing R or r (the radii) should be positive numbers. (Do not evaluate the integral(s)).

3B. Now set up the integral(s) which represents the volume of the solid of revolution obtained by revolving A around the y-axis. (Do not evaluate the integral(s)). All expressions representing R or r (the radii) should be positive numbers.

4. Let I be the exact value of $\int_0^{\pi} (x\cos(2x) + 1)\, dx$ (You are welcome to use your calculator's built-in numerical integration to see that $I = \pi$).

Facts: if $f(x) = x\cos(2x) + 1$ then $\boxed{f'(x) = \cos(2x) - 2x\sin(2x)}$ and $\boxed{f''(x) = -4\sin(2x) - 4x\cos(2x)}$.

4A: Use an appropriate graph (use Ymin = −6 and Ymax = 6 for a nice picture?) on your calculator to estimate the best value of K_1 you can use in "theorem 3" to find the maximum possible error if a LHS is used as an approximation for I. (Give K_1 to the nearest 0.2; eg $K_1 = 8.2$ or $K_1 = 7.6$ or $K_1 = 11.8$ etc; your K_1 must end in one of .0, .2, .4, .6 or .8).

$K_1 =$ ☐

4B: For the correct value of K_1 in (4A), find that maximum error guaranteed by theorem 3, if LHS(1000) is used to approximate I. (Answer using **five** digits after the decimal point). Show the formula you use in your work.

4C: Use an appropriate graph (suggestion: Ymin = −15 and Ymax = 15) to find the value of K_2 you should use in theorem 3 to find an error bound for either a TRAP or MID approximation of I. (Again K_2 to the nearest 0.2).

$K_2 =$ ☐

4D: Use the value of K_2 obtained in (4C) to find the smallest value of n you can choose so that theorem 3 guarantees MID(n) will be within 0.005 of I. *Show all your computations.*

4E: Find MID(n) for the value of n from (4D) to as many places as your calculator displays.

4F: Fact: $I = \pi$ is the exact answer. What is the actual error in using the answer to (4E)? (That is, find $|I - \text{MID}(n)|$ for the right value of n).

5. The following table of velocities $v(t)$ in feet per second at various times t for some moving object was recorded during an experiment:

t	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$v(t)$	270	211	154	103	58	20	-13	-40	-64	-84	-101

5A. What is the physical meaning of $\int_1^4 v(t)\ dt$?

5B. For the integral $\int_1^4 v(t)\ dt$ in (5A), estimate LHS(3), RHS(3), TRAP(3) and MID(3) and SIMP(3). Use only the information available in the table. Show all your work, and clearly label all your answers!

5C. The table suggests that $v(t)$ is an decreasing, concave-up function on [0, 5]. Suppose it is, and there was enough information to find approximations LHS(n), RHS(n), TRAP(n) and MID(n) to $I = \int_0^5 v(t)\ dt$. From smallest to largest, put these numbers in order: I, LHS(n), RHS(n), TRAP(n) and MID(n).

6A. Find $15 \int_{-10}^{8} x\,\sqrt{(1/2)x+5}\,dx$ by the method of substitution, using appropriate notation throughout. Simplify your final answer.

6B. In particular, what are the limits on the integral in (6A) after the appropriate substitution is made?

7. BONUS! (Only do this if you have time). Suppose an artist decides to make a sculpture based on the region A in problem one, and the sculpture will be a solid object having A as its base, and such that cross-sections through the object and parallel to the y-axis are semi-circles. Set up the integral(s) which represent the volume of the resulting solid.