

Directions: Complete all of the following to the best of your ability. If you do not understand a question, please let me know; I may be able to assist you. **SHOW ALL WORK!** You will be graded primarily on the method you use, not your final answer. **GOOD LUCK!**

Name: \_\_\_\_\_

Question	Points Possible	Score
1	12	
2	15	
3	18	
4	20	
5	20	
Total	75	

**DO NOT WRITE IN THE BOXES ABOVE!**

Each question page is followed by a blank page. Use this extra space if needed but be sure to label your work so it can be graded appropriately.

1. Answer the following questions about the definite integral  $I = \int_{0.5}^{0.75} \left(x - \frac{1}{\sqrt{x}}\right)^2 dx$ .

(a) Find the exact value of  $I = \int_{0.5}^{0.75} \left(x - \frac{1}{\sqrt{x}}\right)^2 dx$ .

You can write your final answer using the evaluation bar  $(F(x)) \Big|_{0.5}^{0.75}$ .

(b) The function  $f(x) = \left(x - \frac{1}{\sqrt{x}}\right)^2$  is decreasing and concave up on the interval  $[0.5, 0.75]$ . Which of the following pairs of approximations will give an overestimate of  $I$ ? Circle one correct answer.

$R_n$  and  $M_n$

$L_n$  and  $M_n$

$R_n$  and  $T_n$

$L_n$  and  $T_n$

2. Answer the following questions related to  $\int x^3 \sqrt{x^2 - 1} \, dx$

(a) Evaluate the indefinite integral  $\int x^3 \sqrt{x^2 - 1} \, dx$

(b) For  $I = \int_2^{11} x^3 \sqrt{x^2 - 1} \, dx$  find the following components needed to calculate  $R_{10}$ .

$\Delta x =$  \_\_\_\_\_

$x_k =$  \_\_\_\_\_

$f(x_k) =$  \_\_\_\_\_

3. The following questions are about  $f(x) = x^{1/2} - \frac{x^{3/2}}{3}$

(a) Find a value  $K_1$  that is appropriate for the  $R_n$  error bound formula

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

for  $I = \int_4^9 x^{1/2} - \frac{x^{3/2}}{3} dx$ .

(b) Set up and evaluate the integral for the arc length (not  $I$  of part (a)) for  $f(x) = x^{1/2} - \frac{x^{3/2}}{3}$  over the interval  $4 \leq x \leq 9$ .

You should write your final answer using the evaluation bar notation  $(F(x)) \Big|_4^9$ .

4. The following questions have to do with the curves given by  $y = x^4$  and  $y = 2x^3$ .

(a) Use integrals to write an expression that computes the area for the region between the two curves over the interval  $[1, 4]$ . **DO NOT EVALUATE ANY INTEGRAL!**

(b) Consider the two curves  $y = x^4$  and  $y = 2x^3$  and the enclosed region between where they cross. Write an integral for the volume of the solid made by rotating/revolving this region about the line  $y = -1$ . For partial credit, be sure to identify the appropriate interval, your  $r_o$  and  $r_i$ , and the integrand (inside of the integral). **DO NOT EVALUATE ANY INTEGRAL!**

5. The following questions have to do with the effects of gravity in the Nintendo game *Super Mario Galaxy*. In that game the protagonist Mario explores the galaxy by jumping from planet to planet.

- (a) In some parts of the game, the effect of gravity can be modeled by the Separable Differential Equation

$$y' = y^2(\sin x - e^{2x}).$$

Find an explicit solution to this Differential Equation.

- (b) In other parts of the game *Super Mario Galaxy*, the force exerted when  $x$  units from a black hole is given by

$$F(x) = \frac{6}{x^2 + 1}.$$

Set up and evaluate the integral for the amount of work required for Mario to jump from a planet that is 4 units from the black hole to an above planet that is 8 units from the black hole.

You may write your final answer using the evaluation bar notation.