

Directions: Complete all of the following to the best of your ability. If you do not understand a question, please let me know; I may be able to assist you. **SHOW ALL WORK!** You will be graded primarily on the method you use, not your final answer. **GOOD LUCK!**

Name: _____

Question	Points Possible	Score
1	12	
2	15	
3	18	
4	20	
5	20	
Total	75	

DO NOT WRITE IN THE BOXES ABOVE!

Each question page is followed by a blank page. Use this extra space if needed but be sure to label your work so it can be graded appropriately.

1. Answer the following questions about the definite integral $I = \int_1^4 \left(1 + \frac{1}{x}\right)^2 dx$.

(a) Find the exact value of $I = \int_1^4 \left(1 + \frac{1}{x}\right)^2 dx$.

You can write your final answer using the evaluation bar $(F(x)) \Big|_1^4$.

(b) The function $f(x) = \left(1 + \frac{1}{x}\right)^2$ is increasing and concave up on the interval $[1, 4]$. Which of the following pairs of approximations will give an overestimate of I ? Circle one correct answer.

R_n and M_n

L_n and M_n

R_n and T_n

L_n and T_n

2. Answer the following questions related to $\int \frac{x}{\sqrt[4]{x-1}} dx$

(a) Evaluate the indefinite integral $\int \frac{x}{\sqrt[4]{x-1}} dx$

(b) For $I = \int_3^5 \frac{x}{\sqrt[4]{x-1}} dx$ find the following components needed to calculate L_{10} .

$\Delta x =$ _____

$x_{k-1} =$ _____

$f(x_{k-1}) =$ _____

3. The following questions are about $f(x) = \frac{x^3}{6} + \frac{1}{2x}$

(a) Find a value K_1 that is appropriate for the R_n error bound formula

$$|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$$

for $I = \int_2^3 \frac{x^3}{6} + \frac{1}{2x} dx$.

(b) Set up and evaluate the integral for the arc length (not I of part (a)) for $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ over the interval $2 \leq x \leq 3$.

You should write your final answer using the evaluation bar notation $(F(x)) \Big|_2^3$.

4. The following questions have to do with the curves given by $y = 3x^2$ and $y = x^3$.

(a) Use integrals to write an expression that computes the area for the region between the two curves over the interval $[1, 5]$. **DO NOT EVALUATE ANY INTEGRAL!**

(b) Consider the two curves $y = 3x^2$ and $y = x^3$ and the enclosed region between where they cross. Write an integral for the volume of the solid made by rotating/revolving this region about the line $y = -2$. For partial credit, be sure to identify the appropriate interval, your r_o and r_i , and the integrand (inside of the integral). **DO NOT EVALUATE ANY INTEGRAL!**

5. The following questions have to do with the effects of gravity in the Nintendo game *Super Mario Galaxy*. In that game the protagonist Mario explores the galaxy by jumping from planet to planet.

- (a) In some parts of the game, the effect of gravity can be modeled by the Separable Differential Equation

$$y' = y^4(e^x + \cos 2x).$$

Find an explicit solution to this Differential Equation.

- (b) In other parts of the game *Super Mario Galaxy*, the force exerted when x units from a black hole is given by

$$F(x) = \frac{4}{x^2 + 1}.$$

Set up and evaluate the integral for the amount of work required for Mario to jump from a planet that is 10 units from the black hole to an above planet that is 15 units from the black hole.

You may write your final answer using the evaluation bar notation.