**Math 205 A B** Quiz 02 (remake) page 1 09/26/2012 **Name** Suggested solution

**1.** Suppose an economy has just three sectors $C$, $N$, $S$. Suppose that $C$ consumes 20% of its own output while the rest of $C$'s output is divided equally for consumption by sectors $N$ and $S$. Suppose $N$ consumes 70% of its own product, $S$ consumes 20% of $N$'s output and the remainder of $N$'s output is used by $C$. Finally, of what is produced by $S$, 90% is taken by $N$ and the rest is used by $C$ (so $S$ takes none of its own output).

1A: In the space below, give both the exchange table for this economy *and* the corresponding system of equations that you need to solve in determining the equilibrium prices $P_C$, $P_N$ and $P_S$ for this economy.

| C | N | S | |
|---|---|---|---|
| .2 | .1 | .1 | C |
| .4 | .7 | .9 | N |
| .4 | .2 | 0 | S |

$$\begin{cases} P_C = 0.2P_C + .1P_N + .1P_S \\ P_N = .4P_C + .7P_N + .9P_S \\ P_S = .4P_C + .2P_N \end{cases} \quad \text{-OR-} \quad \begin{cases} 0 = -.8P_C + .1P_N + .1P_S \\ 0 = .4P_C - .3P_N + .9P_S \\ 0 = .4P_C + .2P_N - 1P_S \end{cases}$$

1B: Find the set of equilibrium prices $P_C$, $P_N$ and $P_S$, if given that $P_S = 100$ million dollars. Show any RREF'd matrices involved in your solution.

the corresponding augmented matrix is

$$\left[\begin{array}{ccc|c} -.8 & .1 & .1 & 0 \\ .4 & -.3 & .9 & 0 \\ .4 & .2 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -.6 & 0 \\ 0 & 1 & -3.8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} P_C = .6P_S \\ P_N = 3.8P_S \\ P_S \text{ is free.} \end{cases}$$

Now, given $P_S = \$100M$, we obtain $P_C = \$60M$ and $P_N = \$380M$

1C: Find the set of equilibrium prices $P_C$, $P_N$ and $P_S$, if given that the price $N$ pays $C$ for what $N$ consumes of $C$'s output is 80 million dollars.

we're given that $N$ consumes $.4$ of $C$'s output, so it must pay $.4$ of $P_C$

that is, $.4P_C = \$80M$. Thus $\boxed{P_C = \frac{\$80M}{.4} = \$200M}$

Next, since $P_C = .6P_S$, we learn $\$200M = .6P_S \therefore \boxed{P_S = \$333.\overline{33}M}$

Finally, $P_N = 3.8P_S = 3.8 * \$333.\overline{33}M = \$1266.\overline{66}M$; so $\boxed{P_N = \$1266.\overline{6}M}$

**2.** Let $A = \begin{bmatrix} 4 & 5 & 1 & 3 \\ 4 & 8 & 4 & 4 \\ 4 & -1 & -5 & 3 \\ 1 & -2 & -3 & 1 \end{bmatrix}$ and $\mathbf{s} = \begin{bmatrix} 11 \\ 12 \\ 5 \\ -1 \end{bmatrix}$. It's a fact that RREF$(A|\mathbf{s})$ is $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$.

2A. Find in parametric vector form all solutions to $A\mathbf{x} = \mathbf{s}$.

from RREF$(A|\vec{s})$ we find

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 + x_3 \\ 1 - x_3 \\ x_3 \\ -2 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ 1 \\ 0 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \text{ where } x_3 \text{ is free}}$$

2B. Find a non-trivial solution $\mathbf{x}$ to $A\mathbf{x} = \mathbf{0}$ or explain why there are none.

for $A\vec{x} = \vec{0}$, ALL the sol'n's are given by "$\vec{v}_h$", that is, $x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$ where $x_3$ is free.

Take $x_3 = 1$ (for example) to obtain the non-trivial sol'n $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}$.