

**Math 106: Review for Exam I**

1. Find the following. [Substitution tip: usually let  $u =$  a function that's "inside" another function, especially if  $du$  (possibly off by a multiplying constant) is also present in the integrand.]

(a)  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(b)  $\int_{\pi}^{2\pi} \cos^7(5x) \sin(5x) dx$

(c)  $\int \frac{7x^2}{1+x^6} dx$

(d)  $\int x\sqrt{10-x} dx$

2. Suppose  $f(x)$  is decreasing and concave up.

(a) Put the following quantities in ascending order.

$$L_{100}, R_{100}, T_{100}, M_{100}, \int_a^b f(x) dx$$

(b) What can you say with certainty about where  $S_{200}$  would fit into your list above?

3. Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to  $\int_4^{12} f(t) dt$  given the data in the table below.

$t$	4	6	8	10	12
$f(t)$	15	11	8	4	3

4. Find bounds for each of the following errors if  $I = \int_2^7 \ln x dx$ .

(a)  $|I - L_{100}|$

(b)  $|I - T_{100}|$

(c)  $|I - M_{100}|$

5. If  $I = \int_2^7 \ln x \, dx$ , how many subdivisions are required to obtain a trapezoidal sum approximation with error of at most  $1/1,000,000$ ?

6. Solve the differential equation  $dy/dx = 2xy + 6x$  if the solution passes through  $(0, 5)$ . [Students in Professor Ross's sections should omit this problem.]

7. Write integrals equal to

(a) the arc length of  $y = x^2$  on the interval  $[1, 5]$

(b) the area bounded by  $y = x^2 - 8x + 24$  and  $y = 3x$

8. Consider the region bounded by  $y = \sqrt{x}$ ,  $y = 0$ , and  $x = 9$ . Write an integral equal to the volume generated if this region is revolved about

(a) the  $x$ -axis

(b) the line  $x = -1$

9. A pyramid has a square base 30 feet to a side and a height of 10 feet. Write integrals equal to

(a) the volume of the pyramid

(b) the work done in pumping all the fluid to a point 5 feet above the pyramid if the pyramid is filled to a height of 8 feet with water (which weighs 62.4 pounds per cubic foot)