

MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 2

NAME: _____ Section: (Circle one) A(8 : 00) B(9 : 30)

Show ALL your work CAREFULLY.

Let

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & -1 \\ 8 & 5 & 0 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 11 \\ -3 \\ 2 \end{bmatrix}.$$

(a) Does the matrix equation $A\vec{x} = \vec{b}$ have a solution? If yes, find one.

The corresponding augmented matrix is

$$\begin{bmatrix} 1 & 0 & 4 & 11 \\ 2 & 2 & -1 & -3 \\ 8 & 5 & 0 & 2 \end{bmatrix}.$$

Using elementary row operations, the reduced row echelon form of A can be obtained as follows.

$$\begin{aligned} \begin{bmatrix} 1 & 0 & 4 & 11 \\ 2 & 2 & -1 & -3 \\ 8 & 5 & 0 & 2 \end{bmatrix} &\sim \begin{bmatrix} 1 & 0 & 4 & 11 \\ 0 & 1 & -9 & -25 \\ 8 & 5 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 & 11 \\ 0 & 1 & -9 & -25 \\ 0 & 5 & -32 & -86 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 & 11 \\ 0 & 1 & -9 & -25 \\ 0 & 0 & 13 & 39 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 0 & 4 & 11 \\ 0 & 1 & -9 & -25 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 & 11 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}. \end{aligned}$$

THE solution is $x_1 = -1, x_2 = 2$ and $x_3 = 3$.

(b) Do the columns of A span \mathbb{R}^3 ? Explain.

YES, they do. From part (b), the reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

It follows that for any arbitrary vector \vec{b} , the equation $A\vec{x} = \vec{b}$ always has a (unique) solution. Thus, every \vec{b} is a linear combination of the columns of A .