Math 205 Quiz 2
Name: KEY

1. Consider the following augmented matrix (in ref) for a system of equations written $[A|\vec{b}]$.

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   (a) Write the solution to the system in Parametric Vector Form.

$$\left\{ t \begin{bmatrix} \frac{-3}{2} \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ 2 \\ 0 \end{bmatrix} \mid t \text{ is any real number.} \right\}$$

   (b) Write the solution to homogeneous equation ($A\vec{x} = \vec{0}$) in Parametric Vector Form. (This shouldn't take any additional work.)

$$\left\{ t \begin{bmatrix} \frac{-3}{2} \\ 0 \\ 1 \end{bmatrix} \mid t \text{ is any real number.} \right\}$$

   (c) Although you do not know the columns of the matrix A, do the columns of A form a linearly independent set? If so, explain. If not, find a linear dependence relation.

   No. There is a free variable and infinitely many solutions to the homogeneous equation. $-3\vec{a_1} + 2\vec{a_3} = \vec{0}$ so $2\vec{a_3} = 3\vec{a_1}$ is a dependence relation.

2. Let $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Consider the Span$\{\vec{v}, \vec{h}\}$ where $\vec{h} \in \mathbb{R}^2$.

   What must be true of $\vec{h}$ for the Span$\{\vec{v}, \vec{h}\} = \mathbb{R}^2$?

   We know that $\vec{h}$ must not be a multiple of $\vec{v}$. Any other $\vec{h}$ together with $\vec{v}$ will span $\mathbb{R}^2$.

3. If the equation $A\vec{x} = \vec{0}$ has exactly one solution, then does $A\vec{x} = \vec{b}$ have exactly one solution for every $\vec{b}$? Explain.

   No. If $A\vec{x} = \vec{b}$ has a solution, then there is exactly one solution. However, it is possible that there is not solution to $A\vec{x} = \vec{b}$ if the system is inconsistent. (ie. The columns of A are linearly independent put they don't span the codomain.)

   Consider the following rref(A):

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

   This matrix would have exactly one solution to $A\vec{x} = \vec{0}$ because there are no free variables; however, the columns of A do not span $\mathbb{R}^4$ so there will not be a solution for all $\vec{b}$.