MATH 205A,B - LINEAR ALGEBRA
FALL 2015

QUIZ 1

NAME: **Section:**(Circle one) A(8am) B(9 : 30am)

Show ALL your work CAREFULLY.

Consider the following matrix

$$A = \begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 2 & 0 & 4 \\ 3 & -6 & 9 & 6 \end{bmatrix}.$$

(a) Find the reduced row echelon form of the matrix A. Make sure you show the row operations you use.

The reduced row echelon form of A can be obtained as follows.

$$\begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 2 & 0 & 4 \\ 3 & -6 & 9 & 6 \end{bmatrix} \xrightarrow{\text{(i)}} \begin{bmatrix} 1 & -2 & 3 & 2 \\ -2 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{(ii)}} \begin{bmatrix} 1 & -2 & 3 & 2 \\ 0 & -2 & 6 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{(iii)}} \begin{bmatrix} 1 & -2 & 3 & 2 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{(iv)}} \begin{bmatrix} 1 & 0 & -3 & -6 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where (i) is the row operation *add* $(-3)\cdot$ *(first row) to (third row)***; (ii) is the row operation** *add* $(2)\cdot$ *(first row) to (second row)***; (iii) is the row operation** *multiply (second row) by* $(-1/2)$**; (iv) is the row operation** *add* $(2)\cdot$ *(second row) to (first row)***.**

(b) Find the general solutions of the system whose augmented matrix is given by A.

Based upon the reduced row echelon form of A obtained in part (a), we conclude that

$$x_1 = 3x_3 - 6; x_2 = 3x_3 - 4; x_3 \textbf{ is a free variable } .$$

Note that, if we choose x_2 as a free variable then

$$x_3 = \frac{x_2 + 4}{3} \quad \textbf{and} \quad x_1 = 3\left(\frac{x_2 + 4}{3}\right) - 6.$$

Similarly, we can choose x_1 to be a free variable.

Date: Sept. 14, 2015.