Math 205AB Quiz 01 page 1 09/14/2012 **Name** Suggested solutions

NOTE: If you use the RREF of a matrix to answer any question, be sure to write that matrix down as part of your answer! Use "vector hats", and the word "free" where appropriate.

1. Let $A = \begin{bmatrix} -2 & 0 & 4 & -4 \\ 2 & -3 & -19 & 13 \\ -3 & 2 & 16 & -12 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 11 \\ -3 \\ 11 \end{bmatrix}$, and $\mathbf{z} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}$.

1A) Is **b** in the span of the set of column vectors of A? *Explain!*

The vector $\vec{b}$ is in the span of those columns if and only if $A\vec{x}=\vec{b}$ has a solution. Consider $\text{rref}(A|\vec{b})$ which is $\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & 0 \\ 0 & 1 & 5 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

The last row represents the equation

$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$ which has no solution.

So $A\vec{x}=\vec{b}$ has no soln. So NO, $\vec{b} \notin$ span of those column vectors.

1B) Find the vector $A\mathbf{z}$.

thats the L.C. $0\begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix} + 2\begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 4 \\ -19 \\ 16 \end{bmatrix} + 0\begin{bmatrix} -4 \\ 13 \\ -12 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -6 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ -19 \\ 16 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -25 \\ 20 \end{bmatrix}$

1C) Is the span of the set of column vectors of A equal to $\mathbb{R}^3$? *Explain!*

NO! In part 1A we found a specific example of a vector, namely $\begin{bmatrix} 11 \\ -3 \\ 11 \end{bmatrix}$, which is in $\mathbb{R}^3$ but not in the span of the set of col. vectors of A.

(you can also appeal to "theorem 4", which says, the span is all of $\mathbb{R}^3 \Leftrightarrow$ A has a pivot position in every row (and this is not the case here since rref(A) has a row of 0's))

2. Use the methods and notation developed in class to solve this system of equations:

$$\begin{cases} -2x_1 \quad\quad\quad + 4x_3 - 4x_4 = 8 \\ 2x_1 - 3x_2 - 19x_3 + 13x_4 = 1 \\ -3x_1 + 2x_2 + 16x_3 - 12x_4 = 6 \end{cases}$$

the rref of the corresponding matrix is

$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 2 & -4 \\ 0 & 1 & 5 & -3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, which represents a system having soln's $\begin{cases} x_1 = -4 + 2x_3 - 2x_4 \\ x_2 = -3 - 5x_3 + 3x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$

just FYI, (note this side is the same as in 1A) (but this col. is different)

and these are also the soln's of the given system.