MATH 205: Quiz 1 Show your work.
Name:

1. Consider the following matrix in reduced row echelon form (RREF).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) If the above matrix is a **coefficient matrix** for a system of equations, then what can you say about the solution of the system? (ie. Will it have a solution? One? Infinitely many? Any restrictions on what the augmented column could be?)

   There is a exactly one solution to this system.

   (b) If the above matrix is an **augmented matrix** for a system of equations, then what can you say about the solution of the system?

   The system is inconsistent. It has no solutions.

2. Consider the following vectors.

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

   (a) Give two vectors (besides $\vec{v_1}$ and $\vec{v_2}$) that are in the Span$\{\vec{v_1}, \vec{v_2}\}$. Show your work.

   Any linear combination of $\vec{v_1}$ and $\vec{v_2}$ is in the Span$\{\vec{v_1}, \vec{v_2}\}$. So choose any real scalars c_1 and c_2 and form the vector $c_1\vec{v_1} + c_2\vec{x_2}$. For my examples, I chose $c_1 = c_2 = 1$ and $c_1 = 1$, $c_2 = -1$.

$$\vec{v_1} + \vec{v_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{v_1} - \vec{v_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

   (b) For what values of h is $\vec{y} = \begin{bmatrix} 1 \\ h \\ 4 \end{bmatrix}$ a linear combination of $\vec{v_1}$ and $\vec{v_2}$? Show your work.

   Constructed the following augmented matrix.

$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 2 & 1 & h \\ 0 & 1 & 4 \end{array}\right] \xrightarrow{\text{Switch rows 2 and 3}} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 2 & 1 & h \end{array}\right] \xrightarrow{\mathbf{3=-2*1+3}} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 1 & h-2 \end{array}\right] \xrightarrow{\mathbf{3=3-2}} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & h-6 \end{array}\right]$$

   Therefore, for $\vec{y}$ to be a linear combination of $\vec{v_1}$ and $\vec{v_2}$, the system must be consistent so $h = 6$ is the only value.