

MATH106A CALCULUS II - PROF. P. WONG

FINAL EXAM - APRIL 15, 2015

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	18	
2.	18	
3.	18	
4.	22	
5.	14	
6.	24	
7.	18	
8.	18	
Total	150	

1. Evaluate each of the following integrals.

(9 pts.)(a)

$$\int x^2 \sqrt{x^3 + 1} \, dx$$

(9 pts.)(b)

$$\int \cos x \ln(\sin x) \, dx$$

2. Evaluate each of the following integrals.

(9 pts.)(a)

$$\int \frac{dx}{x(x^2 + 1)}$$

(9 pts.)(b)

$$\int \frac{dx}{x^2 \sqrt{x^2 + 4}}.$$

3.(10 pts.)(a) Use the method of separation of variables to solve the following Initial Value Problem.

$$y' = ye^{-x}, \quad y(0) = 1.$$

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(8 pts.)(b) Consider the following given data of a function $g(x)$ on the interval $[-2, 2]$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2.0
$g(x)$	2	-1	0	-1	1	2	-4	-3	0

Find M_4 (midpoint sum) and R_8 (right-hand sum).

4.(9 pts.) (a) Let R be the region bound by the curves $y = x^2 + 2$ and $y = 4 - x^2$. Find the exact area of the region R . [First sketch a picture of the region.]

(13 pts.) (b) Find the exact volume of the solid formed when the region R (in part (a)) is revolved around the line $y = -3$.

5. (8 pts.)(a) Evaluate the following improper integral if it exists.

$$\int_1^3 \frac{1}{\sqrt{3-x}} dx$$

- (6 pts.)(b) Write a definite integral [Do Not Evaluate] representing the arc length of the graph of $f(x) = \frac{x^2}{4} - \frac{\ln x}{2}$ over the interval $[1, 2e]$. Simplify the definite integral as much as possible.

6. Determine whether each of the following series converges or diverges. **Justify your answer.**

(8 pts.)(a) $\sum_{n=1}^{\infty} \frac{\pi^{7n}}{e^{8n}}$

(8 pts.)(b) $\sum_{n=10}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

(8 pts.)(c) $\sum_{k=1}^{\infty} \frac{k!}{(3k)!}$

7.(8 pts.)(a) Let $f(x) = \frac{1}{1-2x}$. Use the Maclaurin series for $f(x)$ to find $f^{(30)}(0)$.

(10 pts.)(b) Find the Taylor series for $g(x) = \sqrt{x}$ centered at $x_0 = 4$. [Be sure to state the interval of convergence.]

8.(10 pts.)(a) Find the interval of convergence for the following power series. [Don't forget to check the endpoints.]

$$\sum_{n=2}^{\infty} \frac{(2x-3)^n}{n \ln n}$$

(8 pts.)(b) Determine whether the following series converges absolutely, conditionally, or neither.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + 2n}$$