Math 106C
Calculus 2
Final Exam
April 15, 2015

Name________________________________

Mr. Balcomb

Please show your work.

1. Let $I = \int_1^2 \frac{1}{x} dx$.

a) Use the Fundamental Theorem of Calculus to evaluate I exactly.

b) Compute the approximate sum of R_4 . Show your work.

c) Compute the approximation error $|I - R_4|$.

2. Evaluate the following integrals.

a) $\int \frac{1}{x^2+8x+7} dx$

b) $\int sin^2(x)cos^3(x)dx$

3. Let f(x) = ln(1+x)

a) Derive the third degree Taylor polynomial for f based at $x_0 = 0$.

b) Use this polynomial to estimate ln(2).

c) What is the possible error that could have occurred in your estimate in part (b)? Recall that if you use the Taylor polynomial of degree n at x_0 to approximate for x in the interval I containing x_0, then $\frac{K_{n+1}|x-x_0|^{n+1}}{(n+1)!}$ is an upper bound for the approximation error. [K_{n+1} is an upper bound for the absolute value of the (n+1)th derivative of f on I.]

4. Find the volume of the solid generated when the region enclosed by y = tan(x), x = $\pi/4$, and x = 0 is revolved about the x-axis.

5. Find the solution of IVP

$$y' = -\frac{x}{y} \text{ where } y(6) = 8.$$

6. Let f(x) = $\frac{x^2}{2}$. Write the length of this curve from 0 to 1 as an integral (arc length).

7. Do these integrals converge? Justify your answer.

a) $\int_0^\infty (1-x)e^{-x}dx$

b) $\int_0^1 \frac{1}{\sqrt{1-x}}dx$

8. For each of the following series, test to see whether the series converges or diverges and explain why.

a) $\sum_{n=0}^{\infty} \frac{3}{4^n}$

b) $\sum_{k=3}^{\infty} \frac{(2k)!}{4^k}$

c) $\sum_{k=1}^{\infty} (-1)^k \frac{2^k}{k!}$

9. For the series $\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2}$,

a) find the radius of convergence.

b) find the interval of convergence.

10. a) Give the Maclaurin series for cos(x).

Show that this series converges to cos(x) for all x. [Show that the error estimate goes to 0 as the number of terms increases.]
Recall that if you use the Taylor polynomial of degree n at x_0 to approximate for x in the interval I containing x_0, then $\frac{K_{n+1}|x-x_0|^{n+1}}{(n+1)!}$ is an upper bound for the approximation error. [K_{n+1} is an upper bound for the absolute value of the (n+1)th derivative of f on I.]

c) Find a power series expression for xcos(x).

d) Now find a power series expression for $\int xcos(x)dx$

e) Using this formula, approximate $\int_0^1 xcos(x)dx$ with an error less than 0.01. Justify your answer.

11. Evaluate your integral in #6.