NAME____

I___II___III___IV___V___VI___VII___VIII___IX___X___XI___XII___ TOTAL____
(6) (4) (6) (10) (5) (6) (10) (5) (20) (10) (8) (10) (100)

May 14 2010

Mathematics 206
Multivariable Calculus
Late Final Examination

Mr. Haines

(6) I. Give equations for:

A. The set of all points whose distance from (1, 2, 3) is 5.

B. The tangent line to the curve C parametrized by $\boldsymbol{c}(t) = (t, t^2, t^3)$ at the point $\boldsymbol{c}(1)$.

C. The equation of the tangent plane at the point (0, -1, 2) to the surface whose equation is $x^3 + 12y + 3z^2 = 0$.

(4) II. If **a** = **i** + **j** and **b** = **i** - 2**j** **c**ompute:

A. The length of **a** .

B. The dot product of **a** and **b** .

(6) III. If $\boldsymbol{F} : \mathbb{R}^3 \to \mathbb{R}^3$ with rule $\boldsymbol{F}(x, y, z) = (x^3, y^2, z)$.

A. Calculate div $\boldsymbol{F}$

B. Calculate **curl** $\boldsymbol{F}$

(10) IV. For the vector field $\boldsymbol{F}(x, y, z) = (y,\ x,\ 1)$:

A) Give a potential function for **F** .

B) If C is a path in $\mathbb{R}^3$ parametrized by $\boldsymbol{c}(t) = (\cos t, \sin t, 2)$ with $0 \le t \le 2\pi$, calculate the path integral $\int_C \boldsymbol{F} \cdot d\boldsymbol{x}$.

(5) V. Suppose that g : $\mathbb{R}^3 \to \mathbb{R}^1$ with rule $g(x, y, z) = x + y^2 + xz^2$. Calculate the directional derivative of g at the point (1, 1, 2) in the direction that is parallel to the vector **i** + **j** .

(6) VI. Suppose $\boldsymbol{f}(x, y, z) = (x + y,\ xyz^2)$ and **a** = (1, 3, 2).

A) Give the Jacobian matrix of $\boldsymbol{f}$ at **a** .

B) Give the total derivative of $\boldsymbol{f}$ at **a** .

(10) VII. Use the Divergence Theorem to evaluate $\oiint_{\partial S} \mathbf{F} \bullet \mathbf{n}\ d\sigma$, where $\mathbf{F} = x^2\mathbf{i} + xz^2\mathbf{j} + y^2\mathbf{k}$ and ∂S is the surface of the unit cube in the first octant. ($S = [0,1]\times[0,1]\times[0,1]$).

(5) VIII. If $f : \mathbb{R}^3 \to \mathbb{R}$ with rule $f(x, y, z) = 3x^2 + 2xy + 2z$, calculate H$f$(1, 1, 1), the Hessian of f at (1, 1, 1).

(20) IX. Let M be the surface parametrized by

$$\boldsymbol{f}(s,t) = (s, t, 9 - s^3 - t^3);\ 0 \le s \le 1;\ \ 0 \le t \le 2s$$

A. Compute $\frac{\partial \boldsymbol{f}}{\partial s} \times \frac{\partial \boldsymbol{f}}{\partial t}$.

B. Give a unit vector that is perpendicular to M at the point $\boldsymbol{f}(1,1) =$ (1, 1, 7).

C. Set up but do not evaluate an integral which gives the surface area of M.

(continued from the previous page with the same surface M …)

D. For the function $g(x, y, z) = xz$ set up but **do not evaluate** an iterated integral that gives the surface integral of g over M.

E. For the function $\boldsymbol{F}(x, y, z) = (x, y, x + y)$, set up but **do not evaluate** an iterated integral that gives the surface integral of $\boldsymbol{F}$ over M.

(10) X. Evaluate the double integral $\iint_R \sqrt{x^2+y^2}\,dA$ where R is the region in the first quadrant and bounded by the unit circle $x^2+y^2=1$ by converting to polar coordinates. The conversion equations are $x = r\cos\theta\,;\, y = r\sin\theta$.

(8) XI. For the quadratic form $p(x,y,z) = x^2 - 2y^2 + 5z^2 - 2xz$,

A. give a symmetric matrix S that is the matrix of this quadratic form.

B. By taking determinants and using Sylvester's Theorem, determine if p is positive definite, negative definite, indefinite, or none of these.

(10) XII. Evaluate the line integral $\int_C \boldsymbol{F}(x, y, z) \cdot d\boldsymbol{x}$ where $\boldsymbol{F}(x, y, z) = (y, z, x)$ if C is the straight line segment from (0,0,0) to (1,1,1) parametrized by $c(t) = (t, t, t)$ for $0 \leq t \leq 1$.