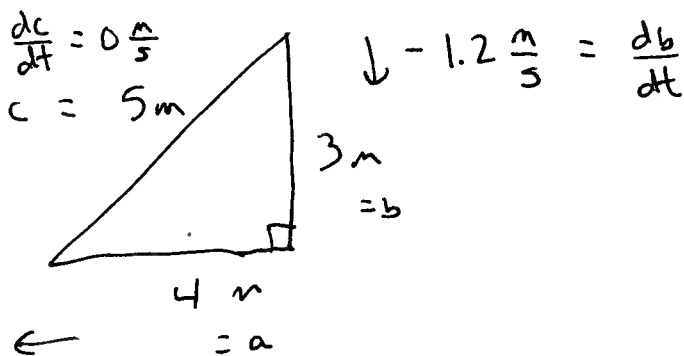


Name: KEY

SHOW ALL WORK, CLEARLY AND LEGIBLY, TO RECEIVE FULL CREDIT. CORRECT SPELLING, ORGANIZATION OF YOUR SOLUTION, AND PROPER USE OF MATHEMATICAL NOTATION ALL COUNT. YOU MAY USE A STAND-ALONE GRAPHING CALCULATOR, BUT NOT ANY INTERNET-BASED CALCULATORS. NO NOTES, BOOKS, OR OTHER ADDITIONAL RESOURCES ARE PERMITTED. GOOD LUCK!

1.) (10 pts.) Suppose a 5-meter ladder is sliding down a wall at a rate of 1.2 meters per second. How fast is the distance from the wall to the ladder's bottom changing when the top of the ladder is 3 meters above the ground?



$$\frac{da}{dt} = ?$$

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2 \cdot 4 \frac{da}{dt} + 2 \cdot 3 \cdot 1.2 = 2 \cdot 5 \cdot 0$$

$$8 \frac{da}{dt} - 7.2 = 0$$

$$\frac{da}{dt} = \frac{7.2}{8}$$

$$\boxed{\frac{da}{dt} = 0.9 \frac{m}{s}}$$

2.) (15 pts.) Consider the integral $\int_0^6 x^2 dx$.

a.) (5 pts.) Use M_3 to compute the value of the integral.



$$\begin{aligned} & 2 \cdot 1^2 + 2 \cdot 3^2 + 2 \cdot 5^2 \\ & = 2 + 18 + 50 \\ & = \boxed{70} \end{aligned}$$

b.) (5 pts.) Use the Fundamental Theorem of Calculus to compute the value of the integral.

$$f(x) = x^2$$

$$F(x) = \frac{1}{3}x^3$$

$$\int_0^6 x^2 dx = \frac{1}{3}(6)^3 - \frac{1}{3}(0)^3 = \frac{1}{3}(216) - \frac{1}{3}(0) = \boxed{72}$$

c.) (5 pts.) Which of the above methods is most accurate? How do you know this? Justify your response with information from our course this semester.

Part (b) is exactly correct, according to the FTC. Part (a) is one of many ways to estimate.

3.) (15 pts.)

- a.) (5 pts.) What does the Intermediate Value Theorem say about whether the function $f(x) = x^3 - 2x + 1$ has a root on the interval $[0, 2]$ of x -values?

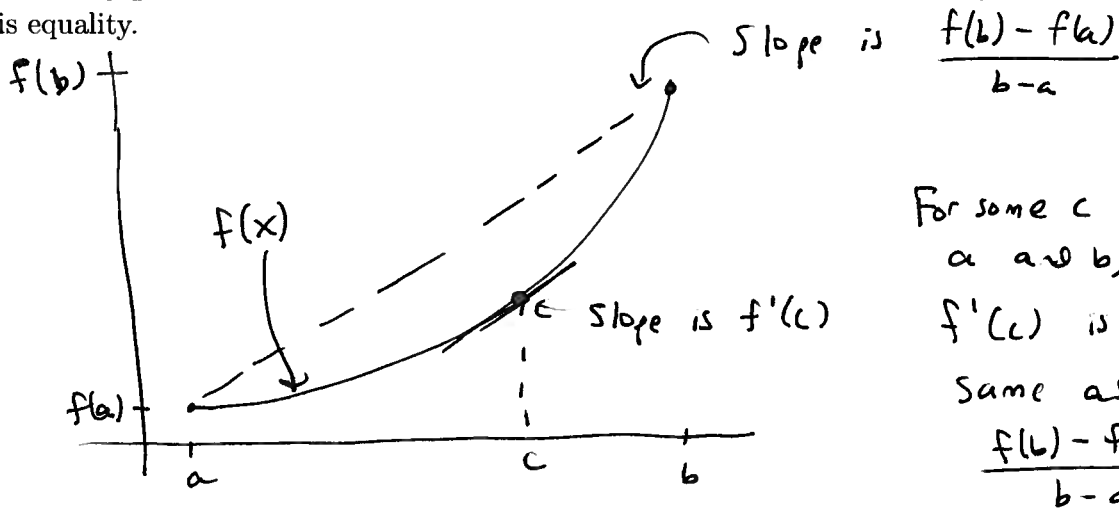
$$f(0) = 1$$

$$f(2) = 8 - 4 + 1 = 5$$

IVT does not imply there is a root of $f(x)$ on $[0, 2]$

(That is: IVT cannot be used to show this.)

- b.) (5 pts.) The conclusion of the Mean Value Theorem (MVT) includes the equality $f'(c) = \frac{f(b)-f(a)}{b-a}$. Draw and label a graph to show what the MVT is referring to with this equality.



- c.) (5 pts.) Expand the sigma notation $\sum_{k=2}^5 (2k+1)$ into a sum. You do not need to simplify further.

$$(2 \cdot 2 + 1) + (2 \cdot 3 + 1) + (2 \cdot 4 + 1) + (2 \cdot 5 + 1)$$

4.) (15 pts.)

a.) (5 pts.) Solve the initial-value problem (IVP)

$$y' = 3 \sin x - 4e^x, \quad y(\pi) = 3.$$

$$y(x) = -3 \cos x - 4e^x + C$$

$$y(\pi) = -3 \cos(\pi) - 4e^\pi + C = 3$$

$$-3(-1) - 4e^\pi + C = 3$$

$$3 - 4e^\pi + C = 3$$

$$\begin{aligned} -4e^\pi + C &= 0 \\ C &= 4e^\pi \\ \hline y &= -3 \cos x - 4e^x + 4e^\pi \end{aligned}$$

b.) (5 pts.) Compute the derivative of $y = \ln(\cos(\arctan(e^x)))$. You do not need to simplify further.

$$y' = \frac{1}{\cos(\arctan(e^x))} \cdot \left[-\sin(\arctan(e^x)) \right] \cdot \frac{1}{1+(e^x)^2} \cdot e^x$$

c.) (5 pts.) Find an antiderivative of $f(x) = \frac{5x^4 - 6x + \sec^2 x}{x^5 - 3x^2 + \tan x}$. Be sure to confirm it is a valid antiderivative.

$$F(x) = \ln(x^5 - 3x^2 + \tan x)$$

$$\text{Check: } F'(x) = \frac{1}{x^5 - 3x^2 + \tan x} \cdot (5x^4 - 6x + \sec^2 x)$$

$$= f(x) \quad \checkmark$$

5.) (10 pts.) Use the table to evaluate the following.

x	10	11
$f(x)$	e	2
$f'(x)$	π	3
$g(x)$	5	-1
$g'(x)$	-2	7

a.) (5 pts.) $h'(10)$ if $h(x) = f(x) \cdot g(x)$

$$h'(x) = f'(x)g(x) + f(x)g'(x)$$

$$h'(10) = (\pi)(5) + (e)(-2)$$

$$h'(10) = 5\pi - 2e$$

b.) (5 pts.) $j'(11)$ if $j(x) = \frac{f(x)}{g(x)}$

$$j'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$j'(11) = \frac{(-1)(3) - (2)(7)}{(-1)^2} = \frac{-3 - 14}{1} = -17$$

6.) (5 pts.) Compute the limit $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin(3x)}$. You may use any algebra- or calculus-based method to do so. You may check your answer with a graph, but a graph alone is insufficient justification for a response and will earn no credit.

As $x \rightarrow 0$:

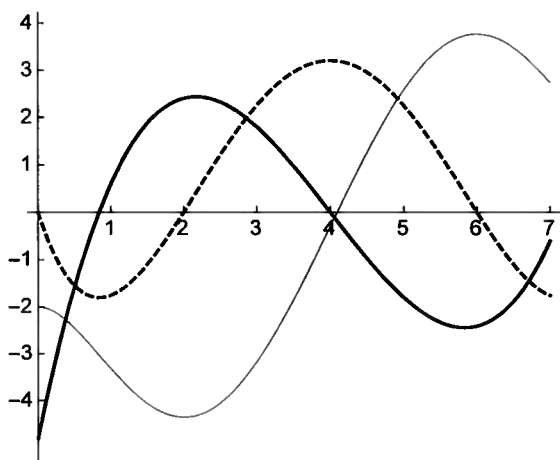
$$1 - \cos x \rightarrow 1 - 1 = 0$$

$$\sin(3x) \rightarrow 0$$

So: L'Hôpital's Rule
will work

$$y = \lim_{x \rightarrow 0} \frac{\sin x}{3 \cos(3x)} = \frac{0}{3} = 0$$

7.) (10 pts.) The graphs below are f , f' , and f'' . State which is which, and state how you know this.



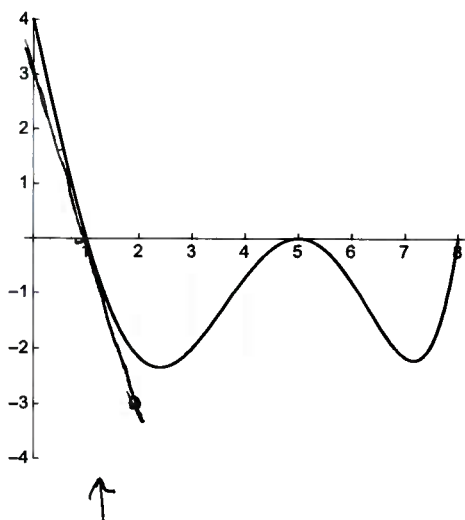
f : thin curve
 f' : dashed curve
 f'' : thick curve

f has stationary points at about $x=2$, $x=6$, and f' has zeroes at these x -values.

Also, f is increasing between $x=2$ and $x=6$, and f' is positive on the same interval.

f' has stationary points at $x=1$ and $x=4$, and f'' has zeroes at these points. (Also: Concavity can be used to compare f with f'')

8.) (5 pts.) The graph below is $f(x)$. Use the grid to estimate the slope of the tangent line to $f(x)$ at $x=1$, and write the equation of this tangent line.



Slope: -3

Point $(1, 0)$

Equation: $0 = -3(1) + b$

$b = 3$

$$y = -3x + 3$$

Slope $\approx \frac{-3}{1} = -3$

9.) (15 pts.) Use the limit definition of the derivative to compute $f'(x)$ if $f(x) = \frac{1}{x} + 3$. You may check your answer with the Power Rule, but the Power Rule by itself earns no credit.

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\left(\frac{1}{x+h} + 3\right) - \left(\frac{1}{x} + 3\right)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{(x) - (x+h)}{x(x+h)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{xh(x+h)}
 \end{aligned}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\
 &= \frac{-1}{x(x)} \\
 &= \boxed{\frac{-1}{x^2}}
 \end{aligned}$$

BONUS (5 pts.): Select a topic from this semester's Calculus I course. Explain that topic to someone who is not currently taking calculus. Write a paragraph about how that explanation/discussion went: what did you say? How did they respond? If they had questions, how did you answer those questions? What were your thoughts on the explanation/discussion as a whole? You may have already handed in this bonus; alternately, you may respond to it on the back of this page.