

Final Exam
MATH 106 - A&B Winter 2016

Version A

Name: _____ Section (A or B): _____

Instructions:

- Answer as many of the following questions as possible.
- No cell phones or collaboration allowed. If you leave the classroom during the exam you must leave your cell phone with the instructor.
- Approved calculators are allowed.
- Additional scrap paper is available upon request.
- *Multiple choice questions:* Circle the letter corresponding to your answer. No partial credit will be awarded.
- *Short answer questions:* Show all of your work on the page of the problem. Clearly indicate your answer and the reasoning that you used to arrive at the answer. You do not have to simplify algebraic expressions.

This exam has 4 multiple choice problems and 6 short answer problems. There are a total of 100 points. Good luck!

Problem	Possible Points	Points Earned
MC	16	
5	12	
6	6	
7	12	
8	20	
9	16	
10	18	
TOTAL	100	

Theorems for Reference

Error bounds for approximating sums. Let $I = \int_a^b f(x) dx$, and let L_n, R_n, T_n , and M_n denote the left, right, trapezoid, and midpoint sums for I , each with n equal subdivisions.

- **Left- and right-rule errors.** Let K_1 be a constant such that $|f'(x)| \leq K_1$ for all x in $[a, b]$. Then

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n}; \quad |I - R_n| \leq \frac{K_1(b-a)^2}{2n}.$$

- **Midpoint- and trapezoid-rule errors.** Let K_2 be a constant such that $|f''(x)| \leq K_2$ for all x in $[a, b]$. Then

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}; \quad |I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}.$$

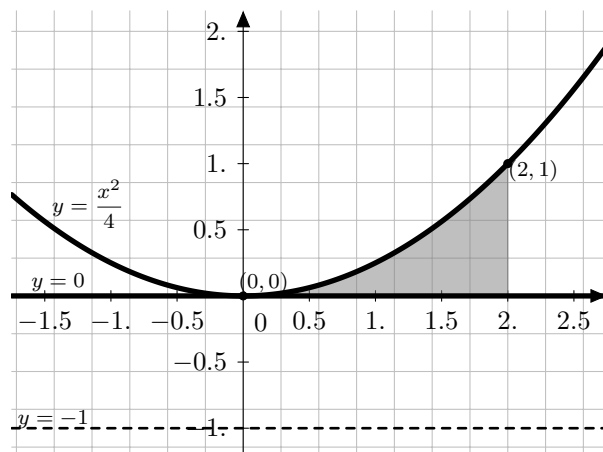
Taylor's theorem. Let $P_n(x)$ be the n th-order Taylor polynomial for $f(x)$ based at x_0 . Suppose that for all x in I ,

$$\left| f^{(n+1)}(x) \right| \leq K_{n+1}.$$

Then

$$|f(x) - P_n(x)| \leq \frac{K_{n+1}}{(n+1)!} |x - x_0|^{n+1}.$$

1. (4 points) Let $I = \int_a^b f(x) dx$ and suppose that $f(x)$ is decreasing and concave up on the interval $[a, b]$. Which of the following statements is true?
- A. $T_n \leq I \leq L_n$
- B. $I \leq L_n \leq R_n$
- C. $L_n \leq I \leq R_n$
- D. $R_n \leq I \leq T_n$
- E. $M_n \leq I \leq R_n$
2. (4 points) Which of the following integrals describes the volume of the solid obtained by revolving the region bounded by $y = \frac{x^2}{4}$, $y = 0$, and $x = 2$ around the line $y = -1$?



- A. $\pi \int_0^2 \left(\frac{x^2}{4} - 1 \right)^2 dx$
- B. $\pi \int_0^2 \left(1 + \frac{x^2}{4} \right)^2 - 1 dx$
- C. $\pi \int_0^1 4 - (2\sqrt{y})^2 dy$
- D. $\pi \int_0^2 \left(1 + \frac{x^2}{4} \right) - 1 dx$
- E. $\pi \int_0^1 1 - \left(1 + \frac{x^2}{4} \right)^2 dx$

3. (4 points) Find $f^{(17)}(0)$ for the function $f(x)$ whose Maclaurin series is given by

$$\sum_{k=0}^{\infty} \frac{2^k x^{4k+1}}{(2k)!}.$$

- A. $\frac{(17!)(16)}{8!}$
- B. $17!$
- C. $\frac{2^{17}}{34!}$
- D. $\frac{16}{8!}$
- E. $-\frac{16}{8!}$

4. (4 points) Choose the statement below that correctly identifies the behavior of the series

$$\sum_{n=2}^{\infty} (-1)^n \frac{\sqrt{n}}{n-1}.$$

- A. The series diverges by the n -th term test.
- B. The series is conditionally convergent by the alternating series test.
- C. The series is absolutely convergent because $\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{n-1} = 0$.
- D. The series diverges by the alternating series test.
- E. The series is absolutely convergent by the ratio test.

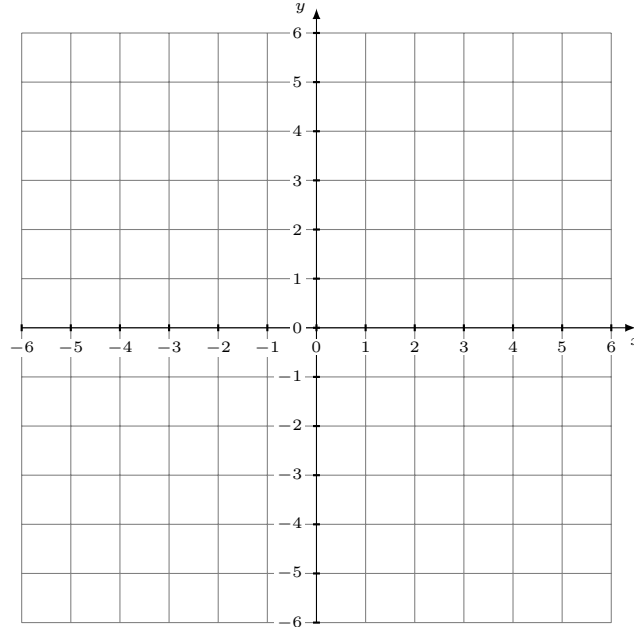
5. (12 points) Compute the following integrals. Do not use Taylor series.

(a) (6 points) $\int \frac{2x - 1}{2x^2 - 2x - 4} dx$

(b) (6 points) $\int \frac{\ln x}{x^2} dx$

6. (6 points)

- (a) (2 points) Draw the region enclosed by the line $y = -2x + 4$ and the parabola $y = -x^2 + 4$ and label the points of intersection.



- (b) (4 points) Set up (**but do not evaluate**) an integral with respect to x that computes the area of the enclosed region. *You do not have to simplify your answer.*

7. (12 points)

- (a) (6 points) Determine whether the improper integral is convergent or divergent. If the integral is convergent, evaluate it. Show all of your work for the integration.

$$\int_1^{\infty} \frac{2x}{(x^2 + 2)^2} dx$$

- (b) (6 points) Use part (a) and the integral test to determine the behavior of the series $\sum_{k=1}^{\infty} \frac{2k}{(k^2 + 2)^2}$. Verify all assumptions of the integral test. If the series converges, find an upper and lower bound for the value of its sum.

8. (20 points) Determine if each of the following series is convergent or divergent. Justify your answers.

(a) (8 points) $\sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3}$

(b) (8 points) $\sum_{n=0}^{\infty} \frac{(-2)^{n+1}}{3^n}$

- (c) (4 points) If possible, find the sum of the series in part (b). If this is not possible, leave this problem blank.

9. (16 points) Find the radius and interval of convergence for the following power series.

(a) (8 points) $\sum_{n=1}^{\infty} \frac{n(x+1)^n}{4^n}$

Radius of convergence: _____ Interval of convergence: _____

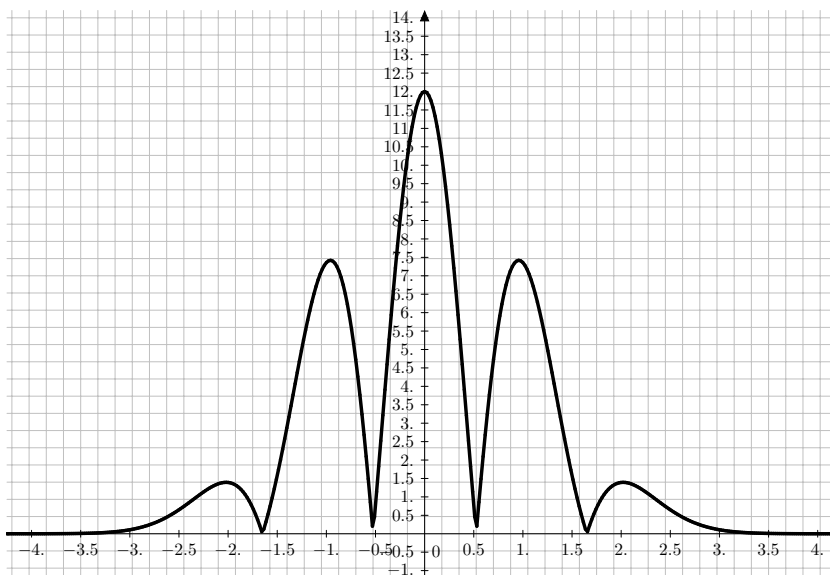
(b) (8 points) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

Radius of convergence: _____ Interval of convergence: _____

10. (18 points) For this problem, consider the function $f(x) = e^{-x^2}$.

(a) (4 points) Find $P_3(x)$, the third-degree Maclaurin polynomial for $f(x)$.

(b) (2 points) What does Taylor's theorem imply about the maximum error committed by P_3 over the interval $[-1, 1]$? A plot of $|f^{(4)}(x)|$ is given below.



(c) (6 points) Find the Maclaurin series for $f(x) = e^{-x^2}$. Write out at least 4 non-zero terms of the series, or write in sigma notation.

(d) (6 points) Evaluate $\int e^{-x^2} dx$ as an infinite series. Write out at least 4 non-zero terms of the series, or write in sigma notation.