Math 105: Review for Final Exam, Part II

1. You are standing on a pier, 6 feet above the deck of a boat. Attached to the boat is a line, which you are pulling in at a rate of 3 feet per second. When there are 10 feet of line between your hand and the boat, at what rate is the boat moving across the water?

2. You are watching a plane flying toward your position at a constant height of 3 miles and a speed of 500 miles per hour relative to the ground. At the moment when the plane is 5 miles from you (diagonally), at what rate is the angle of your vision toward the plane changing?

3. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 2x - 1$ has a root on $[1, 2]$.

4. What (if anything) does the Extreme Value Theorem say about $f(x) = x^2$ on each of the following intervals?

   (a) $[1, 4]$

   (b) $(1, 4)$

5. Find the value of the constant c that the Mean Value Theorem specifies for $f(x) = x^3 + x$ on $[0, 3]$.

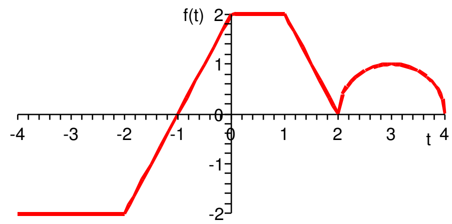
6. Water is leaking out of a tank at a decreasing rate $r(t)$ as shown in the table below.

time (min)	0	2	4	6	8
rate (gal/min)	15	11	8	4	3

   (a) Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

   (b) Interpret the expression $\int_2^6 r(t)\,dt$ in terms of the situation described above.

7. Consider the graph of $f(t)$ shown. It is made of straight lines and a semicircle.



Let $G(x) = \int_0^x f(t)\,dt$ and $H(x) = \int_{-3}^x f(t)\,dt$.

(a) Compute $G(2)$, $G(4)$, $G(-4)$, and $H(4)$.

(b) Where is G increasing? Where is G decreasing?

(c) Where is G concave up? Where is G concave down?

(d) At what x-value(s) does G have a local maximum? At what x-value(s) does G have a local minimum?

(e) Find a formula that relates G and H.

(f) How would your answers to (b), (c), and (d) change if the questions were about H instead of G?

8. (a) Use sigma notation to express L_{10} and M_{10} as approximations to $\int_{20}^{60} \ln x \; dx$.

(b) Draw a sketch that represents the sum M_4.

9. Find the following.

(a) all antiderivatives of $1 + 2x + x^3 + 4\sqrt{x} + \dfrac{1}{x^5} + \sec^2(6x) + \dfrac{7}{1 + 100x^2}$

(b) $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\, dx$

(c) $\displaystyle\frac{d}{dx} \int_{1}^{x} \sin \sqrt{t}\, dt$

(d) $\displaystyle\int_{0}^{2} x^2\, dx$